Loyola University Chicago **Quiz 5**
Math 201, Section 001, Fall 2009

Name (print): ____________________ **Signature:** ____________________

You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

Problem 1. (5 pts) Prove, by induction, that for every $n \in \mathbb{N}$,

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Problem 2. (5 pts) Prove, by induction, that for every $n \in \mathbb{N}$ greater than 1,

$$\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!} < 3 - \frac{2}{(n+1)!}.$$

Problem 3. (6 pts) Let $a_0 = 0$, $a_1 = 1$, and for $n \geq 2$, let $a_n = 2a_{n-1} - a_{n-2} + 2$.

(a) Calculate a_2, a_3, a_4, and a_5.

(b) Predict a formula for a_n based on your calculations.

(c) Prove your prediction from (b).

Problem 4. (4 pts) State the Binomial Theorem and use it to prove that, for all $n \in \mathbb{N}$,

$$\binom{n}{0} + \binom{n}{1}2 + \binom{n}{2}2^2 + \binom{n}{3}2^3 + \cdots + \binom{n}{n-1}2^{n-1} + \binom{n}{n}2^n = 3^n.$$