

Name (print): \_\_\_\_\_ Signature: \_\_\_\_\_

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You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

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**Problem 1.** (4 pts) Prove, by induction, that for every  $n \in \mathbb{N}$ ,

$$\sum_{i=1}^n i(i+1) = \frac{n(n+1)(n+2)}{3}.$$

**Problem 2.** (4 pts) Prove: if  $1 \leq r \leq n$  then

$$\binom{n}{r-1} + \binom{n}{r} = \binom{n+1}{r}.$$

**Problem 3.** (4 pts) Prove, by strong induction, that if

$$a_0 = 5, \ a_1 = -10, \quad a_n = -4(a_{n-1} + a_{n-2}) \text{ for } n = 2, 3, \dots$$

then  $a_n = 5(-2)^n$ .

**Problem 4.** (4 pts) Prove that  $5^n - 4n - 1$  is divisible by 16 for all  $n \in \mathbb{N}$ .

**Problem 5.** (4 pts) Prove that the product of  $r$  consecutive positive integers is divisible by  $r!$ .