Loyola University Chicago **Worksheet 1**
Math 201, Spring 2010

In the "True or False" questions below, provide a proof if the answer is "True" or provide a counterexample if the answer is "False".

1. True or False: $\forall\, x \in \mathbb{R}\ \exists\, y \in \mathbb{R}\ x - y^2 = 1$

Solution: False: take $x = -7$, *then there is no real* y *such that* $-7 = 1 + y^2$ *because* $1 + y^2 > 0$ *for all* $y \in \mathbb{R}$.

2. True or False: $3 \in \{x \in \mathbb{R}\,;\, x^2 + 8 \leq 6x\} \cap \{y \in \mathbb{Z}\,;\, y^4 \leq 128\}$.

Solution: True: $3 \in \{x \in \mathbb{R}\,;\, x^2 + 8 \leq 6x\}$ *because* $3 \in \mathbb{R}$ *and* $3^2+8 \leq 6{\cdot}3$, *and* $3 \in \{y \in \mathbb{Z}\,;\, y^4 \leq 128\}$ *because* $3 \in \mathbb{Z}$ *and* $3^4 \leq 128$.

3. True or False: $P \text{ AND } Q \iff [(P \text{ AND } (\text{NOT } P)) \implies (P \implies (\text{NOT } Q))]$

Solution: False. Use truth table. Another solution: note that $(P\ \ AND\ \ (NOT\ \ P))$ *is always false so the implication* $(P\ \ AND\ \ (NOT\ \ P)) \implies (P \implies (NOT\ \ Q))$ *is always true. But* $P\ \ AND\ \ Q$ *is only true if* P *and* Q *are true.*

4. True or False: For every integer x and y, xy is divisible by 4 if and only if x is even and y is even.

Solution: False: take $x = 4$, $y = 1$. *Then* $xy = 4$ *is divisible by* 4 *but* y *is NOT even.*

5. Express each of the statements below as a logical expression using quantifiers.

There is a smallest positive integer.

Solution:

$$\exists x \in \mathbb{Z}, x > 0\ \forall y \in \mathbb{Z}, y > 0\ \ x \leq y$$

There is no smallest positive real number.

Solution:

$$NOT\ \ (\exists x \in \mathbb{R}, x > 0\ \forall y \in \mathbb{R}, y > 0\ \ x \leq y) \qquad or \qquad \forall x \in \mathbb{R}, x > 0\ \exists y \in \mathbb{R}, y > 0\ \ y < x$$