Loyola University Chicago
Math 201, Spring 2010

Worksheet 6

Problem 1. Consider the set $S = \{a, b, c, d, e\}$ and the following permutations:

$$\rho = \begin{pmatrix} a & b & c & d & e \\ b & c & d & e & a \end{pmatrix}, \qquad \sigma = \begin{pmatrix} a & b & c & d & e \\ c & a & e & d & b \end{pmatrix}.$$

(a) find ρ^{-1}

(b) find ρ^2

(c) find $\rho \circ \sigma$

(d) find n such that ρ^n is the identity permutation

Problem 2. Find bijections

(a) from $[0,1)$ to $[0,1/2) \cup (1/2,1]$.

Solution: there are many solutions. Two of them are below:

$$f(x) = \begin{cases} x & \text{if} \quad x \neq 1/2 \\ 1 & \text{if} \quad x = 1/2 \end{cases} \qquad g(x) = \begin{cases} x & \text{if} \quad x < 1/2 \\ 3/2 - x & \text{if} \quad x \geq 1/2 \end{cases}$$

(b) from $\left\{0, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$ to $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$

Solution: just match up the elements that are in the same spot in line; in other words, let the bijection ϕ be given as

$$\phi(0) = \frac{1}{2}, \quad \phi\left(\frac{1}{2}\right) = \left(\frac{1}{3}\right), \quad \phi\left(\frac{1}{3}\right) = \left(\frac{1}{4}\right), \ldots$$

In other words,

$$\phi(x) = \begin{cases} \frac{1}{2} & \text{if} \quad x = 0 \\ \frac{1}{n+1} & \text{if} \quad x = \frac{1}{n}, n = 2, 3, \ldots \end{cases}$$

(c) from $[0,1]$ to $(0,1]$

Solution: this gets a tad weird. We will use the function ϕ from part (b) above. Let f on $[0,1]$ be defined as

$$f(x) = \begin{cases} \phi(x) & \text{if} \quad x \in \left\{0, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\} \\ x & \text{otherwise} \end{cases}$$

(d) from $(0,1]$ to $[0,1]$

Solution: in (c) above, we got a bijection from $[0,1]$ to $(0,1]$. Here, we can use its inverse, which amounts to the following:

$$g(x) = \begin{cases} 0 & \text{if} \quad x = \frac{1}{2} \\ \frac{1}{n} & \text{if} \quad x = \frac{1}{n+1}, n = 2, 3, \ldots \\ x & \text{otherwise} \end{cases}$$