# Class Notes on Nonlinear Regression (Chapter 5)

## Class 1

- **Nonlinear models often result from compartmental models (scientific "common sense"), and the parameters are usually very important and interpretable (as compared with linear models)**
- **Need to give starting values, and that often requires understanding the model function and sometimes some ingenuity (p.4)**
- **Iterate to a solution using e.g. MGN (Modified Gauss Newton) method (p.6); results in parameter estimates, and then interpretation or prediction**
- **Rival model functions exist for the same dataset – e.g., $SE_2$ (Simple Exponential), $MM_2$ (Michaelis-Menton), and Lansky model functions all look very similar (like the figure at the bottom of p.9)**
- **CI's: two types are: Wald (estimate +/- t*SE) is based on a parabolic approximation to the SSE or likelihood, and Likelihood-based. PLCI's are often asymmetric, which makes more sense since usually our information about a parameter is asymmetric. Best to use PLCI's, but they can be hard to find. The difference between WCI's (Wald) and PLCI's depends upon "curvature"; more on this later**
- **Better understanding of $MM_2$ model function parameters, and how to give good starting values**
- **Example 5.1 BOD – pp. 7-13: parameter estimation, WCR for θ, LBCR for θ, PLCI's for individual parameters (graph**

**pp.11-12), WCI's for individual parameters from a parabolic approximation – summarized in Tables on p.12**

- **Example 5.2 – linear model, but nonlinear model is appropriate since we are interested in the intra-class correlation (p.14), which is a nonlinear function of the linear model parameters. Find the PLCI from graph on p.14; $\hat{\phi}$ = 0.811 occurs where this plot hits its minimum**

**Class 2**

- **Example 5.3 (Laetisaric acid) another linear model 'reparameterized' into a nonlinear one; here too Wald & Likelihood intervals differ: use PLCI's when available**
- **Example 5.4 – two treatment groups ("conv" vs. "eshb") fitting a 3-parameter curve to each and testing for common parameters. Compound hypothesis (on p.20) is tested using the Full-and-Reduced F statistic,**

$$F_{2,18} = [(0.2465-0.1737)/2] / [0.1737/18] = 3.772,$$

**Here, p-value = 0.0428. What is the conclusion here?**

- **Example 5.5 – downward SE2 doesn't fit (see residuals on p.22), but SE2 with a lag (i.e., a "variable knot") does fit: 95% WCI for knot extends from 25.16 minutes to 46.19 minutes; what is the interpretation?**
- **Example 5.6 – another lag example**
- **Example 5.7 – Fitting one (modified) $LL_4$ model function for May and one for June; wish to test**

$$H_0: \theta_{1M} = \theta_{1J}, \theta_{2M} = \theta_{2J} \text{ and } \theta_{3M} = \theta_{3J}$$

**This is tested using Full-and-Reduced F statistic,**

$$F_{3,24} = [(0.0206-0.0179)/3] / [0.0179/24] = 1.20,$$

**Here, p = 0.329. We retain the claim of common upper and lower asymptotes and slopes for May and June.**

**<u>Class 3</u>**

- **All our models so far are homoskedastic normal NLINs, but data in Example 5.8 show non-constant variance. Letting "rhs" denote the (mean) model function, we propose that $VAR = \sigma^2 * mean^\rho$, where $\rho$ is an additional parameter to be estimated. The case where $\rho = 0$ is then constant variances across the X values. To test $H_0$: $\rho = 0$, we use Wald or LR. Wald gives $t_{55} = 1.4707/0.4699 = 3.13$ and p = 0.0028. More reliable is the LR test $\chi^2 = 254.0 - 245.3 = 8.7$ and p = 0.0032. (That Wald gives a similar p-value means quadratic approx. is good here.) Regardless, we reject the null, and accept heteroskedasticity. One of the consequences is that the SE for the $LD_{50}$ drops from 0.3805 to 0.3297 (drops by 13.4%).**
- **Next, exponential family (generalized) nonlinear models**
- **Example 5.10: return to Menarche example but with the $LD_{50} = \gamma$ as a new model parameter; now, SAS gives a 95% WCI for $\gamma$ in the NLMIXED output. We could also find a PLCI, which would be more reliable**
- **Return to Budworms example in Example 5.11 – we accept common slopes using the $-2\Delta LL$ $\chi^2$ test (p = 0.1797 on p.33)**

- Grauer Logistic curve doesn't fit (see residuals on p.34) when using x = age at death. Output 5.10c indicates that we should use the log-age scale, and new model is Equation (5.25). Then, $LD_{50}$ is estimated as 10.9717 yrs., which is down from 15.7003. Also, note the associated SE drops from 3.9748 to 2.1488, a drop of about 46%.