

## Class Notes for Chapter 6

### Class One

- Relative potency (ratio of two [Normal or otherwise] means) is a nonlinear model – need to use techniques of Chapter 5 here
- **Direct Assay** (pp. 1-9) versus **Indirect Assay** (pp. 9-16)
- Direct Assay examples (6.1, 6.2, and 6.3); Indirect Assay examples (6.4 and 6.5)
- [Example 6.1](#) – ratio of two independent Poisson means (since these are COUNT data) using NLMIXED procedure (p.3). RP of SOAP ( $n_1 = 8$ ) to CONTROL ( $n_2 = 6$ ) is estimated to be 0.6028 and Wald TS testing equal potency is on the bottom of p.3. SAS implies this TS  $\sim t_{14}$  (some would argue  $\sim t_{12}$  or  $\sim z$ ). Likelihood test REDUCED model is fit on p.4 (is this right?):  $\chi_1^2 = 28.2$ ,  $p < 0.0001$ . What is our conclusion here?
- [Example 6.2](#) (pp.4-6):  $y$  = sodium excretion rate SER for two treatment groups, NORMAL ( $n_1 = 7$ ) and B10AE ( $n_2 = 7$ ) – since original data are R skewed, use log-transformed data
- If  $Y_2 = \theta_2 Y_1$ , then  $\log(Y_2) = \log(\theta_2) + \log(Y_1)$ .  $Y_1$  is conc. of substance 1, etc. Now, let  $Z_1 = \log(Y_1)$ , and assume  $Z_1 \sim \text{Normal}(\nu_1, \sigma^2)$ , etc. for substance 2. Plots of the Z's given look more Normal with constant variance.
- New mean relationship is in Eqn. 6.6 and fit in the NLIN which produces Output 6.2a. Now, 95% WCI for true RP is suspect, so we do the Likelihood test, and get  $p = 0.0211$ .
- [Example 6.3](#) –  $Y$  = prostate size for  $n_1 = 5$  CONTROL and  $n_2 = 5$  ESTRADIOL animals. Plot on p.7: data look Normal (symmetric) but variance is not constant. Let's model variances too! If  $Y_2 = \theta_2 Y_1$ , then  $\mu_2 = \theta_2 \mu_1$  and  $\sigma_2^2 = \theta_2^2 \sigma_1^2$ . This is kind of like the Seefeldt example (5.8) from last class. See NLMIXED program on pp.7-8 – why can we **not** use NLIN here? 95% WCI for RP is

(1.84,5.00). Profile likelihood curve is on p.9 with cut-lines at 90% (bottom line), 95% (middle) and 99% (top). From 95% cut line and really good eyes, 95% PLCI is (2.19,5.34). Conclusion: we're 95% confident that Estradiol is at least 2.19 times as potent as Control; since 1 is not in the PLCI, Estradiol is significantly more potent than Control.

## Class Two:

- For **Indirect Assays**, we cannot measure amounts directly, but must make inferences indirectly. We'll fit dose-response curves such as the Binary Logistic or other nonlinear model function. When we do, we usually assess RP (relative potency) by the ratio of the  $LD_{50}$ 's for the two treatments.
- [Example 6.4](#) compares two peptides, Neurotensin (N) and Somatostatin (S) using Binary Logistic models. Note the chosen design here: either 0.01 and then multiplied by  $10^k$  or 0.03 and then multiplied by  $10^k$ . Looking at the graph on p.10, looks like the doses don't go high enough.
- First step: Had to decide which scale to use – jump forward to Box-Cox transformation Eqn. 6.14 on p.27: when  $\theta_6$  is near 0 (as is the case here), then use log-dose.
- Now look at the program on p.11, and write down the explicit formula for  $\pi$  (success probability).
- The first NLMIXED here has unequal slope parameters ( $\theta_3$ ) and the second one (Reduced one) has a common slope: -2LL's are given in the outputs. Here, we retain the assumption of common slopes (test stat:  $\chi_1^2 = 2.4$ ,  $p = 0.1213$ ).
- Then, RP is estimated to be 5.6639
- Which peptide is more potent?
- As to CI's look at Reduced model output (Output 6.4b) on p.12: 95% WCI, (-1.89,13.2) looks weird. How so?

- Profile likelihood plot on top of p.13. Really good eyesight confirms that 95% PLCI for  $\rho$  is (1.59,19.59). Interpretation is after the graph. Consequence/ramification are ...?
- [Example 6.5](#) on p.13 gives a Normal example with the modified MM2 model function in Eqn. 6.9, where  $\theta_1$  is the upper asymptote but what is  $\theta_2$  here? Testing for common upper asymptotes – programs on pp.14-15, and here we do the Full-and-Reduced F test bottom of p.15 (accept same upper asymptote).
- Reduced model is in Output 6.5c, and RP is estimated to be  $\approx 24$ , so standard insulin is approximately 24 times more potent than the A1-B29 insulin variety.

### Class Three:

- We can assess interaction (synergy or antagonism) using either one of the Finney models or the SR model
- The Finney models combine two x's (e.g., doses of two drugs) in the effective dose formula (Equation 6.10) first, and then relates this z (effective dose) to the response variable using either Equation 6.11 or 6.12 or some variant of these
- $\theta_5$  is the key (so-called coefficient of synergy) parameter, with
  - $\theta_5 > 0$  indicating synergy
  - $\theta_5 < 0$  indicating antagonism
  - $\theta_5 = 0$  indicating independent action
- As noted last class, Equation 6.12 is the binary logistic model function using the **log-dose scale** – in practice, one needs to determine which exact scale to use and modify accordingly
- [Example 6.6](#). Gerig 2 phenolic acids (ferulic and vanillic acids) in 3 chambers (blocks). Chosen design in graph on p.19 (six support points, only one of which is an interior point). NLIN output on p.20 indicates significant **antagonism**, but Likelihood

(Full and Reduced test) gives marginal proof: p-value = 0.0254. Clearly need a better study! See the **isobole** on p.19.

- [Example 6.7](#). Upjohn drugs A and B binomial example – design in graph on p.22 (plus additional support points).  $n_k$  mice given a given combination of A and B, and  $y_k$  = number that die is counted; log-scale is indicated (output not shown). These data indicate significant **synergy** between drugs A and B (p.23).
- [Example 6.8](#). Carter ethanol and chloral hydrate binomial; checkerboard design on p.24. Maybe a “Ray Design” would be better. Evidence here of synergy (p = 0.0151).
- [Example 6.9](#). Machado & Robinson. Y = RT activity (counts). Drugs are AZT and DDI. Ray design on p.25 with **3 interior rays**. Normal fit produces conclusion of independent action and the residual plot on p.25 – Yikes! Refit using Poisson distribution and modelling variance – got similar results, so former is on p.26. Conclude significant synergy between these two drugs.
- [Example 6.10](#). Chou and Talalay example shows the need for the Box-Cox scale parameter ( $\theta_6$ ) since its estimate is neither zero (log-dose) nor one (dose) here. Also, response variable here is a fraction, so we take logit transformation to (hopefully) achieve Normality. Then, we observe significant synergy.

#### Class Four

- Sometimes the Finney models are not rich enough and we need a larger model such as the Separate Ray (SR) model. The SR model allows for e.g. synergy for one ray, independent action for another, and antagonism for yet a third. Note for example that the for the Finney model to fit, the slopes must be equal and the LD<sub>50</sub>'s must line up on an **isobole** as on p.19 or

p.22 – the point being that it is a rather ‘narrow’ or restrictive model (that said, it does fit in many cases).

- Lots of notation in the SR model, but the big picture is graph on p.29. Point C is the LD<sub>50</sub> for Drug B and point E is LD<sub>50</sub> for Drug A. Rays 3 ... J ... R are **interior rays** – corresponding to different proportions of drugs A and B (with “**slopes**”  $c_k$  in Equation 6.15). For Ray J, if the LD<sub>50</sub> is at the point D, then we have independent action. If it’s closer to the origin, we have synergy (further from the origin → antagonism). A measure of the actual LD<sub>50</sub> to the one expected under independent action is the **combination index** ( $\kappa_r$ ) for each interior ray. The SR model simultaneously fits separate logistic (or otherwise) curves along each of the rays, and calculates the  $\kappa_r$ ’s.

$\kappa_r = 1 \rightarrow$ <b>independent action</b>
$\kappa_r < 1 \rightarrow$ <b>synergy</b>
$\kappa_r > 1 \rightarrow$ <b>antagonism</b>

- It can be shown that if all the **slope parameters** ( $\theta_3$ ’s) are equal and the  $\kappa_r$ ’s follow a specific algebraic relation, then the SR reduces to the Finney model.
- **Example 6.11.** Martin. On pp.30-31, just one interior ray. Six design points on the interior ray, and 5 on the two exterior rays. Point A is the LD<sub>50</sub> for Deguelin, point B is LD<sub>50</sub> for Rotenone, point C is intersection with interior ray, and point F (filled circle) is the actual LD<sub>50</sub> along the interior ray, so  $\hat{\kappa}_3 = 0.6615$ . Note that Output 6.10a here is better than 6.10b (equal slopes) for these data ( $p = 0.0042$ ). Wald test of  $H_0: \kappa = 1$  is on p.31 – better yet, using the program near the bottom of p.32, likelihood  $-2\Delta LL$  test gives  $\chi_1^2 = 14.3$ ,  $p = 0.0002$ . Finally, since RP estimate is  $\hat{\rho} = 2.6405$ , the interior ray corresponds to the effective fraction  $\hat{f} = 0.6053$  (Equation 6.19) on p.29.

- [Example 6.12.](#) Additional Binomial examples with one interior ray. Hewlett and Plackett DDT and  $\gamma$ -BHC again. Output 6.11a shows that log-dose and dose scales are wrong for these data – see Equations 6.13 and 6.14. Stay on this new scale for these data. Then, can accept equal slopes ( $\chi^2_2 = 0.8$ ), but not independent action – synergy detected here too;  $\hat{\kappa}_3 = 0.4555$ .
- [Example 6.13.](#) Shelton data: response variable here is a fraction, and transformed to Normality with the logit transformation – one interior ray here. Cannot accept common slopes – see Full & Reduced F on p.35, so the Finney model will not fit these data. Synergy detect here  $\hat{\kappa}_3 = 0.4286$  and  $c = \frac{1}{4} \rightarrow \hat{f} = 0.2605$ , which may be too low. See Equation 6.19 on p.29. This example points out that we need a good estimate of  $\rho$  = relative potency before we choose the slope of the ray(s),  $c$ .
- [Example 6.9 continued.](#) Finney model even with the Poisson distribution doesn't fit well – residual plot on p.36 looks possibly wavy. Separate Ray model (with Poisson dist.) fits better – see p.37. This dataset has 3 interior rays with slopes  $c = 10, 5$ , and  $1$ . Synergy is detected along each ray, and we accept a common combination index; test that it equals 1 is rejected ( $\chi^2_1 = 218.9, p < 0.0001$ ). Relative potency estimate is such that these interior rays correspond to the effective fractions  $f = 0.1588, 0.2741$ , and  $0.6537$ .
- [Example 6.14.](#) Goldin cancer example – three interior rays with slopes  $c = 7.5, 1$ , and  $1/7.5$ . Graph on p.38. Independent action along first ray, marginal synergy along the central ray, and strong synergy along gentle-sloped ray. Combination indices can be related to effective fractions as in plot on p.40.