Class Notes on Nonlinear Regression (Chapter 5)

Reminders:

- **Nonlinear materials on 6th, 8th and 13th April**
- **Quiz 3 (on nonlinear) on Tuesday 20th April**
- **Homework 11 (on nonlinear) due on Thursday 8th**
- **Bioassay (applied nonlinear) on 15th April**

Tuesday 4/06 Class

- **Nonlinear models often result from compartmental models (scientific "common sense"), and the parameters are usually very important and interpretable (as compared with linear models)**
- **Need to give starting values, and that requires understanding the model function and sometimes some ingenuity (p.4)**
- **Iterate to a solution using e.g. MGN method; results in parameter estimates, and then interpretation or prediction**
- **Rival model functions exist for the same dataset – e.g., SE2, MM2 (Michaelis-Menton), and Lansky model functions all look very similar (like the Figure at the bottom of p.7)**
- **CI's: two types: Wald ("estimate +/- t*SE") is based on a parabolic approximation to the SSE or likelihood, and Likelihood-based. PLCI's are often asymmetric, which makes more sense since usually our information about a parameter is asymmetric. Best to use PLCI's, but they are a pain to find. The difference between WCI's (Wald) and PLCI's depends upon 'curvature'**

- **Better understanding of MM2 model function parms, and how to give good starting values**
- **<u>Example 5.1 BOD</u> – pp. 7-11: parameter estimation, WCR for θ, LBCR for θ, PLCI's for individual parameters (graph p.10 bottom), WCI's for individual parameters from a parabolic approximation – recapped in Tables on p.12**
- **<u>Example 5.2</u> – linear model, but nonlinear model is appropriate since we are interested in the intra-class correlation (p.13), which is a nonlinear function of the linear model parameters. Find the PLCI from the graph on p.12 bottom; $\hat{\phi}$ = 0.811 occurs where this plot hits its maximum**
- **<u>Example 5.3 (Laetisaric acid)</u> another linear model 'reparameterized' into a nonlinear one; here again, Wald and Likelihood intervals really do differ – use PLCI's when available**
- **<u>Example 5.4</u> – two treatment groups (conv vs. eshb) fitting a 3-parameter curve to each and testing for common parameters. Compound hypothesis (bottom of p.17) is tested using the Full-and-Reduced F statistic,**

  $$F_{2,18} = [(0.2465-0.1737)/2] / [0.1737/18] = 3.772,$$

  Here, p-value = 0.0428. What is our conclusion here?

Thursday 4/08 Class

- **Ex. 5.5 – downward SE2 doesn't fit (see residuals on p.20), but SE2 with a lag ("variable knot") does: 95% WCI for knot goes from 25.16 minutes to 46.19 minutes**
- **Ex. 5.6 – another lag example**
- **Ex. 5.7 – Fitting a (modified) LL4 model function for May and one for June; wish to test**

  H_0: $\theta_{1M} = \theta_{1J}$, $\theta_{2M} = \theta_{2J}$ and $\theta_{3M} = \theta_{3J}$;

  tested using Full-and-Reduced F statistic,

  $$F_{3,24} = [(0.0206\text{-}0.0179)/3] \,/\, [0.0179/24] = 1.20,$$

  Here, p = 0.329. We retain the claim of common upper and lower asymptotes and slopes for M and J.
- **All our models so far are homoskedastic normal NLINs, but data in Ex. 5.8 show non-constant variance. Letting "rhs" denote the (mean) model function, we propose that VAR = σ^2*rhs$^\rho$, where ρ is an additional parameter to be estimated. The case where $\rho = 0$ is then constant variances across the X values. To test H_0: $\rho = 0$, we use Wald or LR. Wald gives $t_{55} = 1.4707/0.4699 = 3.13$ and p = 0.0028. More reliable is the LR test $\chi^2 = 254.0 – 245.3 = 8.7$ and p = 0.0032. (That Wald gives a similar p-value means quadratic approx. is good here.) Regardless, we reject the null, and accept heteroskedasticity. One of the ramifications is that the SE for the LD_{50} drops from 0.3805 to 0.3297 (drops by 13.4%).**

Tuesday 4/13 Class

- **Today, exponential family (and non-Gaussian) nonlinear models**
- **Example 5.10: return to Menarche example but with LD50 = γ as a new model parameter; now, SAS gives a 95% WCI for γ in the NLMIXED output. We could also find a PLCI, which would be more reliable**
- **Return to Budworms example in Example 5.11 – we accept common slopes using the –2ΔLL χ^2 test ($p = 0.1797$ on p.30)**
- **Grauer Logistic curve doesn't fit (see residuals on p.31) when using x = age at death. Output 5.10c indicates that we should use the log-age scale, and new model is Equation (5.25). Then, LD50 is estimated as 10.9717 years.**