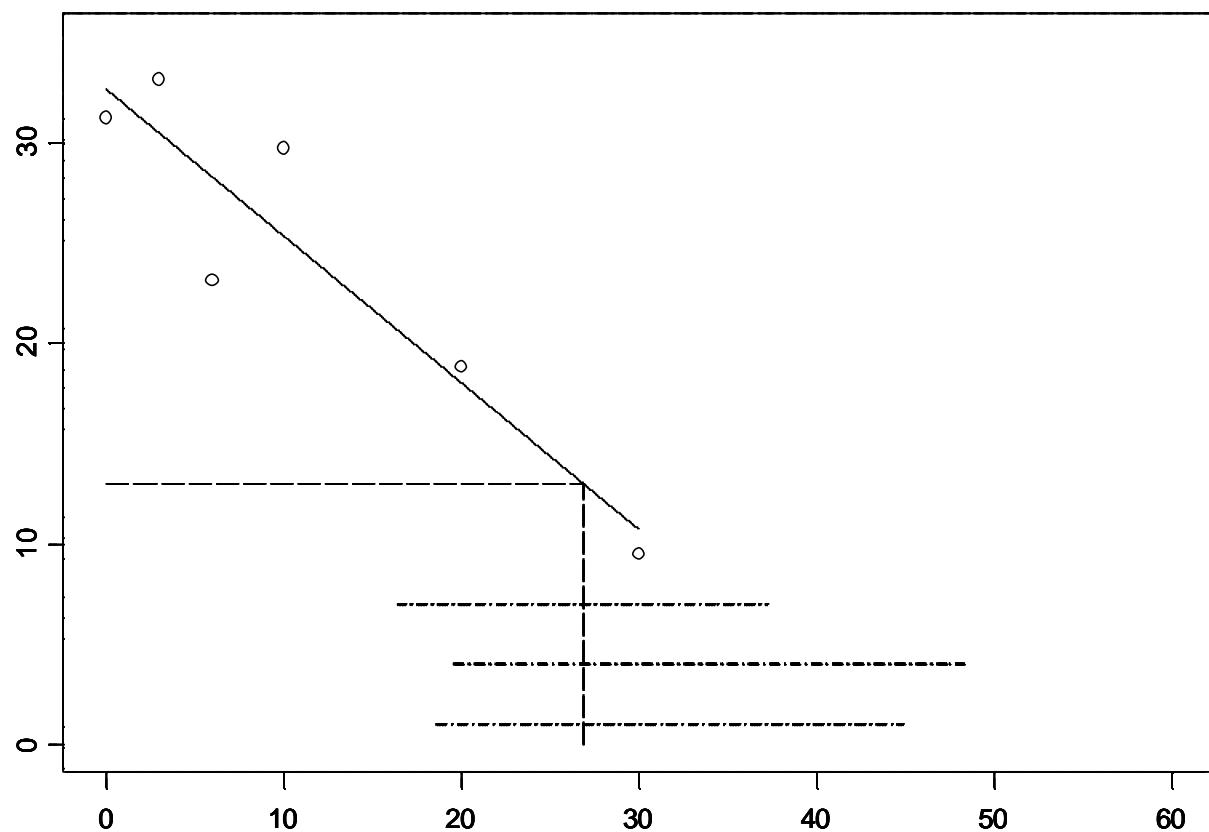


**A. Laetisic Acid – p.7 example 2 in Chapter 5**Model Function:  $\eta(x) = 13 + \beta(x - \gamma_{13})$ 

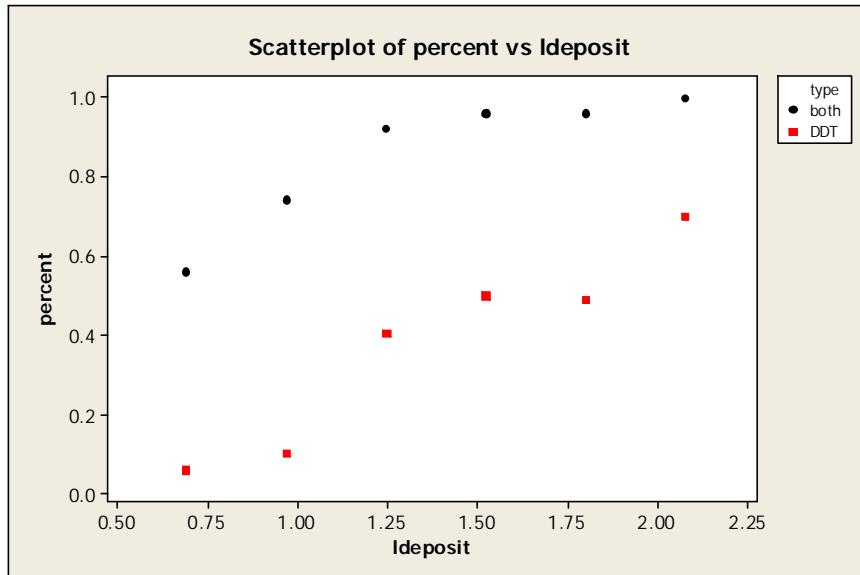
Fungal Growth versus Concentration of Laetisic Acid

**95% Confidence Intervals for  $\gamma_{13}$** 

|      |       |       |
|------|-------|-------|
| Wald | 16.45 | 37.29 |
| PLCI | 19.50 | 48.36 |
| MCCI | 18.53 | 44.88 |

## B. Additional Comments on Dummy Variables

### Homework 4, Exercise 4a



| type | ldeposit | dead | N  | percent | typeboth | TB*LDep |
|------|----------|------|----|---------|----------|---------|
| DDT  | 0.69315  | 3    | 50 | 0.06000 | 0        | 0.00000 |
| DDT  | 0.97078  | 5    | 49 | 0.10204 | 0        | 0.00000 |
| DDT  | 1.24703  | 19   | 47 | 0.40426 | 0        | 0.00000 |
| DDT  | 1.52388  | 19   | 38 | 0.50000 | 0        | 0.00000 |
| DDT  | 1.80171  | 24   | 49 | 0.48980 | 0        | 0.00000 |
| DDT  | 2.07944  | 35   | 50 | 0.70000 | 0        | 0.00000 |
| both | 0.69315  | 28   | 50 | 0.56000 | 1        | 0.69315 |
| both | 0.97078  | 37   | 50 | 0.74000 | 1        | 0.97078 |
| both | 1.24703  | 46   | 50 | 0.92000 | 1        | 1.24703 |
| both | 1.52388  | 48   | 50 | 0.96000 | 1        | 1.52388 |
| both | 1.80171  | 48   | 50 | 0.96000 | 1        | 1.80171 |
| both | 2.07944  | 50   | 50 | 1.00000 | 1        | 2.07944 |

For simplicity, let X denote “ldeposit”, T denote the “typeboth” dummy variable, and TX denote the product,  $TB*LDep = typeboth*ldeposit$ .

### model function

$$\log[\pi/(1-\pi)] = \alpha + \beta*X + \delta*T + \phi*TX, \text{ so the RHS is equal to}$$

$$\alpha + \beta*X \quad \text{for those receiving DDT, and equal to}$$

$$(\alpha + \delta) + (\beta + \phi)*X \quad \text{for those receiving both DDT and g-BHC}$$

It follows that since **d is the difference of the intercepts**, and since **f is the difference of the slopes**, we test whether one curve fits both groups (that is, whether the intercepts and slopes are the same) by testing

$$H_0: \delta = \phi = 0$$

Versus

$$H_A: \text{either } \delta \text{ or } \phi \text{ is non-zero or both.}$$

In the previous exercise, there are two treatment groups, and a dummy variable is created (called “typeboth”) which is unity for one of the groups (BOTH) and zero for the other group (DDT). In contrast, the strategy used in Example 2.6 of Chapter 5 (see p.11) is to create a parameter for each of the two groups. The groups here are the months of May and June, and the dummy variables are  $D_M$  (which is unity for May and zero for June) and  $D_J$  (which is unity for June and zero for May), although these are created in the NLIN procedure with the commands “(month=5)” and “(month=6)” respectively.

It follows that in the latter example, the parameters are

- $\theta_{1M}$  and  $\theta_{1J}$  (the lower asymptotes),
- $\theta_{2M}$  and  $\theta_{2J}$  (the upper asymptotes),
- $\theta_{3M}$  and  $\theta_{3J}$  (the slopes), and
- $\theta_{4M}$  and  $\theta_{4J}$  (the LD<sub>50</sub>'s).

Notice that in contrast with the previous example, here no parameter represents the *difference* of parameters.

If instead, we let  $\theta_{1M}$  be the lower asymptote for May and  $(\theta_{1M} + \delta)$  be the lower asymptote for June, then we would test for equal lower asymptotes by testing

$$H_0: \delta = 0.$$

Using the current method (with  $\theta_{1M}$  for May and  $\theta_{1J}$  for June), we test for equal lower asymptotes by testing

$$H_0: \theta_{1M} = \theta_{1J}$$

Of course, we get the same results, but t-tests can be used using the previous method (over), whereas here we must use (equivalent) Full and Reduced F tests.

## C. SE1 Example to illustrate curvature

Model Function:  $\eta(x) = \exp(-\theta*x)$

Data:  $(x,y) = (0.5,0.93), (4.0,0.025)$  graphed on top of next page with fitted curve

### SAS program

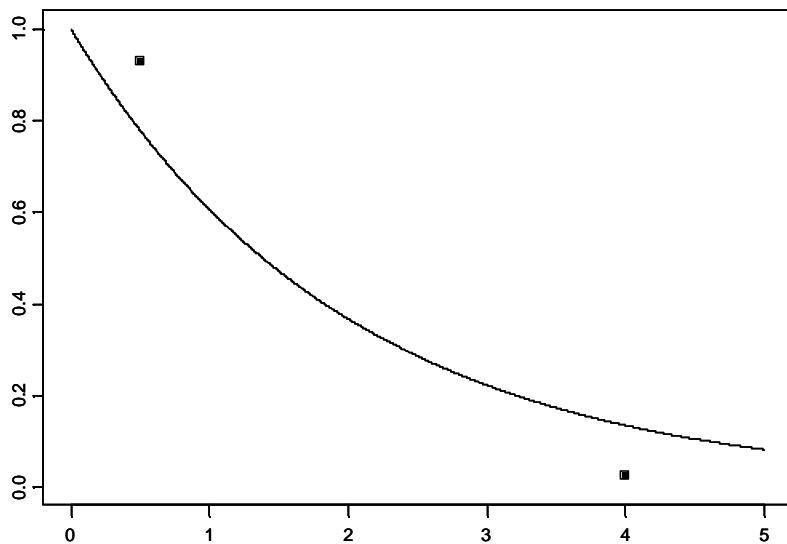
```
data one;
  do x=0.5,4;
    input y @@; output;
  end; datalines;
0.93 0.025
;
proc nlin;
  parms th=20;
  model y=exp(-th*x);
run;
```

### SAS output

| The NLIN Procedure      Dependent Variable y |              |                |             |                       |               |
|--|--------------|----------------|-------------|-----------------------|---------------|
| Method: Gauss-Newton                         |              |                |             |                       |               |
| Iterative Phase                              |              |                |             |                       |               |
| Iter   | th           | Sum of Squares |             |                       |               |
| 0  | 20.0000      | 0.8654         |             |                       |               |
| 1  | 9.9982       | 0.8530         |             |                       |               |
| 2  | 9.6595       | 0.8507         |             |                       |               |
| 3  | 8.2347       | 0.8355         |             |                       |               |
| 4  | 4.7958       | 0.7047         |             |                       |               |
| 5  | 1.8818       | 0.2919         |             |                       |               |
| 6  | 0.9224       | 0.0897         |             |                       |               |
| 7  | 0.5356       | 0.0357         |             |                       |               |
| 8  | 0.4866       | 0.0352         |             |                       |               |
| 20   | 0.5013       | 0.0350         |             |                       |               |
| NOTE: Convergence criterion met.             |              |                |             |                       |               |
| Estimation Summary                           |              |                |             |                       |               |
| Method                                       | Gauss-Newton |                |             |                       |               |
| Iterations                                   | 20           |                |             |                       |               |
| Subiterations                                | 14           |                |             |                       |               |
| Average Subiterations                        | 0.7          |                |             |                       |               |
| Source                                       | DF           | Sum of Squares | Mean Square | F Value               | Approx Pr > F |
| Model  | 1            | 0.8305         | 0.8305      | 23.71                 | 0.1290        |
| Error  | 1            | 0.0350         | 0.0350      |                       |               |
| Uncorrected Total                            | 2            | 0.8655         |             |                       |               |
| Approx                                       |              |                |             |                       |               |
| Parameter                                    | Estimate     | Std Error      | Approximate | 95% Confidence Limits |               |
| th   | 0.5013       | 0.2817         | -3.0782     | 4.0808                |               |

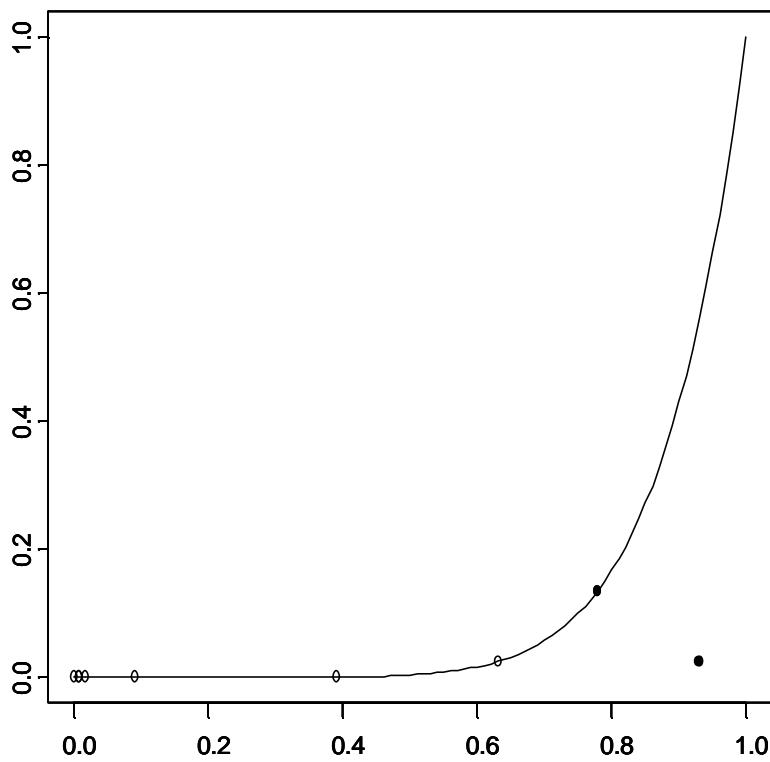
### Data and fitted curve (model function)

Fitted SE1 curve with theta=0.5013



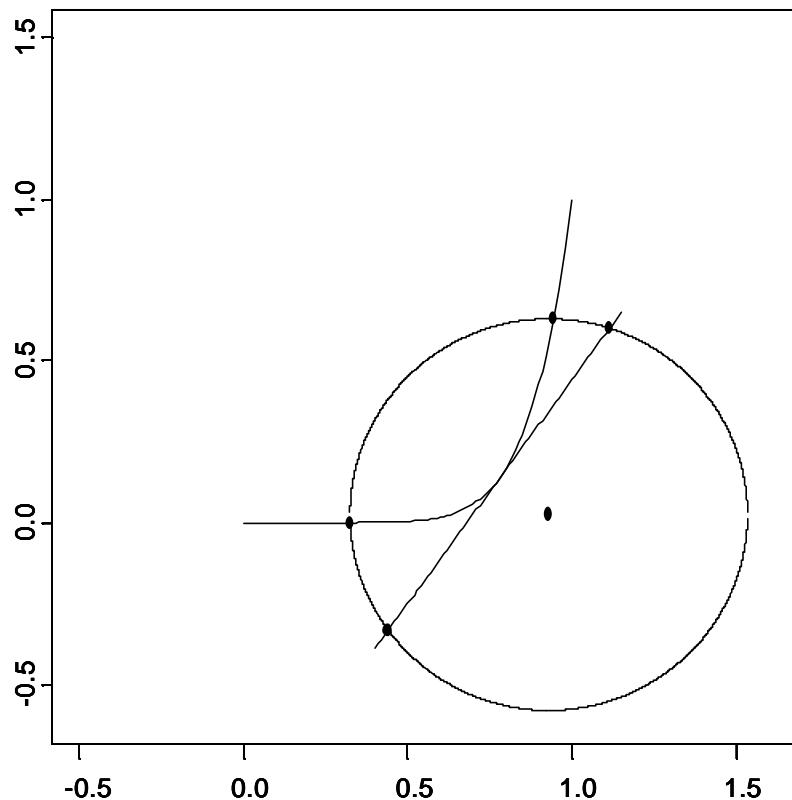
### Graph of Expectation Surface (ES) with data (y) – point on ES corresponds to LSE

One-dim'l expectation surface in 2-dim sample

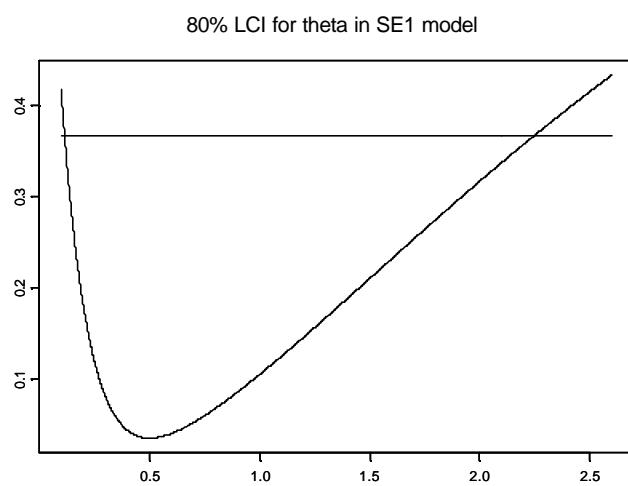


## Graphs of Expectation Surface, tangent plane approximation, data, and 80% CR in Sample Space

SE1: Expectation surface, tangent plane



## Graph of Likelihood Curve and 80% Cut Line



## D. Return to Ex. 2.11 (p.17) – Budworms (Generalized Nonlinear Model)

Are the two curves “parallel”?

### Full Model

```

data one;
do dose=1,2,4,8,16,32;
do sex='male ','female';
  l2d=log2(dose); fem=(sex='female'); male=(sex='male ');
  input knockout @@; n=20; output;
end; end; datalines;
1 0 4 2 9 6 13 10 18 12 20 16
;
proc nlmixed;
parms bmale=1 bfem=1 gmale=6 gfem=6;
l2gmale=log2(gmale); l2gfem=log2(gfem);
b=bmale*male+bfem*fem; l2g=l2gmale*male+l2gfem*fem;
eta=b*(l2d-l2g); ex=exp(eta); pi=ex/(1+ex);
model knockout~binomial(n,pi);
run;

```

### **Output 2.9a. The NL MIXED Procedure (Parameter Estimates)**

-2 Log Likelihood = 35.1

| Parameter | Estimate | Error  | DF | t Value | Pr >  t | Alpha | Lower  | Upper   |
|-----------|----------|--------|----|---------|---------|-------|--------|---------|
| bmale     | 1.2589   | 0.2121 | 12 | 5.94    | <.0001  | 0.05  | 0.7969 | 1.7210  |
| bfem      | 0.9060   | 0.1671 | 12 | 5.42    | 0.0002  | 0.05  | 0.5420 | 1.2701  |
| gmale     | 4.7201   | 0.6674 | 12 | 7.07    | <.0001  | 0.05  | 3.2660 | 6.1742  |
| gfem      | 9.8765   | 1.7819 | 12 | 5.54    | 0.0001  | 0.05  | 5.9941 | 13.7588 |

### Reduced Model

```

proc nlmixed;
parms b=1 gmale=6 gfem=6;
l2gmale=log2(gmale); l2gfem=log2(gfem);
l2g=l2gmale*male+l2gfem*fem;
eta=b*(l2d-l2g); ex=exp(eta); pi=ex/(1+ex);
model knockout~binomial(n,pi);
run;

```

### **Output 2.9b. The NL MIXED Procedure (Parameter Estimates)**

-2 Log Likelihood = 36.9

| Parameter | Estimate | Error  | DF | t Value | Pr >  t | Alpha | Lower  | Upper   |
|-----------|----------|--------|----|---------|---------|-------|--------|---------|
| b         | 1.0642   | 0.1311 | 12 | 8.12    | <.0001  | 0.05  | 0.7786 | 1.3498  |
| gmale     | 4.6889   | 0.7344 | 12 | 6.38    | <.0001  | 0.05  | 3.0888 | 6.2891  |
| gfem      | 9.6037   | 1.5294 | 12 | 6.28    | <.0001  | 0.05  | 6.2714 | 12.9360 |