```
proc sort data=sitka89; by treat; run;
proc nlin;
  title 'OUTPUT # ONE - IN TWO PARTS';
  parms th1=6.6 th2=5.5 th3=100 th4=2;
  t=(timea/th3)**th4; den=1+t;
  model size=th1+(th2-th1)/den;
  by treat;
run;
```

```
                    OUTPUT # ONE - IN TWO PARTS
---------------------------------------- treat=control ----------------------------------------
                                The NLIN Procedure
                              Dependent Variable size
                                Method: Gauss-Newton

                                 Iterative Phase
                                                                          Sum of
              Iter         th1         th2         th3         th4      Squares
                 0      6.6000      5.5000       100.0      2.0000      83.8436
                 1      6.2376      5.8557     46.6649      3.1723      76.5408
                 2      6.5303      5.8451     87.9066      1.1000      74.3248
                 3      6.4359      5.8472     56.6283      1.5931      72.9665
                 4      6.5354      5.8522     80.3989      2.2320      71.1623
                 5      6.4550      5.8572     66.7770      3.5349      70.8937
                 6      6.5259      5.8468     76.4954      3.3817      70.5866
                 7      6.5191      5.8497     75.8201      3.7231      70.5755
                 8      6.5234      5.8490     76.1776      3.6670      70.5751
                 9      6.5228      5.8491     76.1429      3.6825      70.5751
                10      6.5229      5.8491     76.1539      3.6795      70.5751
                11      6.5229      5.8491     76.1519      3.6802      70.5751
              NOTE: Convergence criterion met.

                              Estimation Summary
                        Method                     Gauss-Newton
                        Iterations                           11
                        Subiterations                         2
                        Average Subiterations          0.181818
                        R                              3.85E-6
                        PPC(th4)                       0.000037
                        RPC(th4)                       0.000176
                        Object                         2.62E-10
                        Objective                      70.57508
                        Observations Read                   200
                        Observations Used                   200
                        Observations Missing                  0

                                       Sum of        Mean                Approx
          Source                 DF     Squares      Square    F Value    Pr > F
          Model                   3     13.4823      4.4941      12.48    <.0001
          Error                 196     70.5751      0.3601
          Corrected Total       199     84.0574

                                          Approx
                Parameter      Estimate    Std Error    Approximate 95% Confidence Limits
                th1              6.5229       0.1270      6.2725      6.7733
                th2              5.8491       0.0938      5.6641      6.0341
                th3             76.1519      15.6440     45.2994       107.0
                th4              3.6802       2.7533     -1.7497      9.1100

                           Approximate Correlation Matrix
                              th1             th2             th3             th4
             th1       1.0000000      -0.2146040       0.5409742      -0.7493181
             th2      -0.2146040       1.0000000       0.3649317       0.4038885
```

```
            th3       0.5409742       0.3649317       1.0000000       -0.1509402
            th4      -0.7493181       0.4038885      -0.1509402        1.0000000


-------------------------------------------- treat=ozone --------------------------------------------
                                          The NLIN Procedure
                                       Dependent Variable size
                                         Method: Gauss-Newton
                                           Iterative Phase
                                                                                   Sum of
              Iter           th1           th2           th3           th4        Squares
                 0        6.6000        5.5000         100.0        2.0000          191.4
                 1        6.2270        5.5091       72.9979        2.6349          184.6
                 2        6.1447        5.5193       75.2943        3.6833          183.9
                 3        6.1453        5.5132       76.4836        4.3057          183.8
                 4        6.1480        5.5122       76.6229        4.3331          183.8
                 5        6.1481        5.5122       76.6288        4.3339          183.8
              NOTE: Convergence criterion met.

                                         Estimation Summary
                            Method                         Gauss-Newton
                            Iterations                                5
                            Subiterations                             1
                            Average Subiterations                   0.2
                            R                                  1.218E-6
                            PPC(th4)                            9.21E-6
                            RPC(th4)                           0.000179
                            Object                             1.818E-9
                            Objective                          183.7777
                            Observations Read                       432
                            Observations Used                       432
                            Observations Missing                      0

                                                  Sum of          Mean                   Approx
          Source                        DF       Squares        Square     F Value       Pr > F
          Model                          3       28.0164        9.3388       21.75       <.0001
          Error                        428         183.8        0.4294
          Corrected Total              431         211.8

                                                      Approx
               Parameter         Estimate        Std Error     Approximate 95% Confidence Limits
               th1                 6.1481           0.0769        5.9969        6.2992
               th2                 5.5122           0.0665        5.3814        5.6429
               th3                76.6288          10.4802       56.0294       97.2283
               th4                 4.3339           2.3369       -0.2594        8.9271

                                   Approximate Correlation Matrix
                                th1               th2              th3              th4
            th1       1.0000000      -0.1563266       0.4798255      -0.6691812
            th2      -0.1563266       1.0000000       0.4020466       0.3617470
            th3       0.4798255       0.4020466       1.0000000      -0.0567718
            th4      -0.6691812       0.3617470      -0.0567718       1.0000000
```

```
proc nlmixed data=sitka89;
  title 'OUTPUT # TWO - IN TWO PARTS';
  parms th1=6.6 th2=5.5 th3=100 th4=3.5 se2=.5 sa2=.1;
  th1a=th1+a; th2a=th2+a;
  t=(timea/th3)**th4; den=1+t;
  mean=th1a+(th2a-th1a)/den;
  model size~normal(mean,se2);
  random a~normal(0,sa2) subject=tree;
  by treat;
run;
```

OUTPUT # TWO - IN TWO PARTS

The NLMIXED Procedure
Specifications

Data Set	WORK.SITKA89
Dependent Variable	size
Distribution for Dependent Variable	Normal
Random Effects	a
Distribution for Random Effects	Normal
Subject Variable	tree
Optimization Technique	Dual Quasi-Newton
Integration Method	Adaptive Gaussian Quadrature

---------- treat=control ----------

The NLMIXED Procedure
Dimensions

Observations Used	200
Observations Not Used	0
Total Observations	200
Subjects	25
Max Obs Per Subject	8
Parameters	6
Quadrature Points	1

Parameters

th1	th2	th3	th4	se2	sa2	NegLogLike
6.6	5.5	100	3.5	0.5	0.1	163.145736

Iteration History

Iter	Calls	NegLogLike	Diff	MaxGrad	Slope
1	17	-30.801828	193.9476	745.7998	-1202.27
2	20	-31.547921	0.746093	3003.932	-1459
3	22	-45.409706	13.86178	13599	-6618.3
4	23	-66.625903	21.2162	4674.464	-170.244
5	25	-83.243888	16.61798	323.0923	-14.5484
6	28	-87.94317	4.699282	423.865	-17.4755
7	31	-88.418642	0.475472	317.7789	-1.68586
8	34	-88.641091	0.222449	263.1412	-0.47892
9	36	-88.718472	0.077381	77.70022	-0.04824
10	38	-88.752405	0.033933	55.00256	-0.04286
11	39	-88.786626	0.034222	126.893	-0.01928
12	40	-88.84235	0.055724	75.01125	-0.03106
13	42	-89.265021	0.422671	107.1187	-0.10002
14	44	-91.693732	2.428711	167.4893	-0.78878
15	46	-96.993665	5.299933	523.3232	-20.0808
16	48	-99.760056	2.766391	86.93456	-4.01777
17	50	-101.69521	1.93515	235.1448	-2.60113
18	52	-102.8457	1.150493	129.223	-1.38274
19	54	-103.03005	0.184346	32.22693	-0.40368
20	56	-103.05489	0.024843	3.64833	-0.06105
21	58	-103.06139	0.006498	8.690945	-0.01921
22	60	-103.06163	0.000245	1.888753	-0.00048
23	62	-103.06164	5.557E-6	0.68562	-0.00001
24	64	-103.06164	4.794E-7	0.031779	-1.01E-6

NOTE: GCONV convergence criterion satisfied.

Fit Statistics

-2 Log Likelihood	-206.1
AIC (smaller is better)	-194.1
AICC (smaller is better)	-193.7

BIC (smaller is better)	-186.8

Parameter Estimates

Parameter	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper	Gradient
th1	6.5229	0.1188	24	54.92	<.0001	0.05	6.2778	6.7680	0.001077
th2	5.8491	0.1181	24	49.51	<.0001	0.05	5.6053	6.0929	-0.00108
th3	76.1522	2.6475	24	28.76	<.0001	0.05	70.6881	81.6163	-6.51E-6
th4	3.6801	0.4346	24	8.47	<.0001	0.05	2.7831	4.5770	0.000028
se2	0.01040	0.001112	24	9.35	<.0001	0.05	0.008108	0.01270	0.031779
sa2	0.3425	0.09724	24	3.52	0.0017	0.05	0.1418	0.5432	0.000707

---------- treat=ozone ----------

The NLMIXED Procedure

Dimensions

Observations Used	432
Observations Not Used	0
Total Observations	432
Subjects	54
Max Obs Per Subject	8
Parameters	6
Quadrature Points	1

Parameters

th1	th2	th3	th4	se2	sa2	NegLogLike
6.6	5.5	100	3.5	0.5	0.1	358.899776

Iteration History

Iter	Calls	NegLogLike	Diff	MaxGrad	Slope
1	17	-61.383351	420.2831	1679.045	-2871.64
2	20	-65.975734	4.592382	5467.574	-3969.72
3	25	-136.11056	70.13483	10699.49	-22368
4	26	-166.40417	30.2936	1075.219	-177.528
5	27	-183.47868	17.07451	2700.369	-30.7964
6	31	-201.97775	18.49907	292.4281	-15.6818
7	33	-203.57479	1.597039	1081.804	-8.39846
8	35	-204.30171	0.726924	344.198	-2.83292
9	37	-204.43735	0.135637	96.42194	-0.34232
10	39	-204.50446	0.067114	21.55013	-0.06258
11	41	-204.64837	0.143905	368.3592	-0.05164
12	43	-205.4199	0.77153	852.0323	-0.20037
13	45	-223.03312	17.61322	3015.569	-1.14381
14	47	-225.95203	2.918913	2674.129	-13.1372
15	49	-231.94532	5.993285	524.422	-24.1811
16	50	-239.58031	7.634992	2758.293	-10.5421
17	53	-243.41435	3.834046	386.686	-7.92542
18	55	-244.82299	1.408635	245.8723	-3.07731
19	57	-245.49493	0.67194	341.0889	-1.15289
20	58	-246.32692	0.831994	85.44519	-0.91571
21	60	-246.41534	0.088417	105.4742	-0.22185
22	62	-246.41968	0.004343	44.00521	-0.01874
23	64	-246.42214	0.00246	1.447079	-0.00505
24	66	-246.42217	0.000021	0.560322	-0.00005
25	68	-246.42217	3.554E-7	0.001625	-6.54E-7

NOTE: GCONV convergence criterion satisfied.

Fit Statistics

-2 Log Likelihood	-492.8
AIC (smaller is better)	-480.8
AICC (smaller is better)	-480.6
BIC (smaller is better)	-468.9

Parameter Estimates

Parameter	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper	Gradient
th1	6.1481	0.08849	53	69.48	<.0001	0.05	5.9706	6.3256	-0.00163
th2	5.5122	0.08835	53	62.39	<.0001	0.05	5.3350	5.6894	0.001574
th3	76.6290	1.5106	53	50.73	<.0001	0.05	73.5990	79.6589	-0.00001
th4	4.3339	0.3317	53	13.07	<.0001	0.05	3.6686	4.9992	-0.00017
se2	0.008920	0.000649	53	13.75	<.0001	0.05	0.007619	0.01022	-0.00131
sa2	0.4165	0.08037	53	5.18	<.0001	0.05	0.2553	0.5777	0.000108

```
proc nlmixed data=sitka89;
  title 'OUTPUT # THREE';
  parms th1=6.6 th1add=-0.4 th2=5.8 th2add=-0.3 th3=76 th3add=0.5
    th4=3.7 th4add=0.6 se2=.01 sa2=.4;
  th1a=th1+th1add*dumoz+a;
  th2a=th2+th2add*dumoz+a;
  th3a=th3+th3add*dumoz;
  th4a=th4+th4add*dumoz;
  t=(timea/th3a)**th4a; den=1+t;
  mean=th1a+(th2a-th1a)/den;
  model size~normal(mean,se2);
  random a~normal(0,sa2) subject=tree;
run;
```

OUTPUT # THREE

The NLMIXED Procedure

Specifications

Data Set	WORK.SITKA89
Dependent Variable	size
Distribution for Dependent Variable	Normal
Random Effects	a
Distribution for Random Effects	Normal
Subject Variable	tree
Optimization Technique	Dual Quasi-Newton
Integration Method	Adaptive Gaussian Quadrature

Dimensions

Observations Used	632
Observations Not Used	0
Total Observations	632
Subjects	79
Max Obs Per Subject	8
Parameters	10
Quadrature Points	1

Parameters

th1	th1add	th2	th2add	th3	th3add	th4	th4add	se2
6.6	-0.4	5.8	-0.3	76	0.5	3.7	0.6	0.01

Parameters

sa2	NegLogLike
0.4	-310.12067

Iteration History

Iter	Calls	NegLogLike	Diff	MaxGrad	Slope
1	8	-311.49868	1.378008	1184.523	-61206.9
2	12	-337.98513	26.48645	2380.004	-18783.8
3	14	-346.36227	8.377135	3326.276	-532.635
4	15	-348.2115	1.849228	1103.246	-6.24668
5	16	-348.57757	0.366071	216.6432	-0.68588
6	19	-348.59569	0.01812	63.0535	-0.07629
7	21	-348.59766	0.001976	8.61477	-0.00397

8	23	-348.59842	0.000761	13.82029	-0.00066
9	24	-348.59973	0.001306	2.974258	-0.00107
10	26	-348.60045	0.000718	4.960959	-0.00089
11	28	-348.60415	0.003703	0.681405	-0.00051
12	30	-348.60447	0.000322	3.642538	-0.00012
13	31	-348.6047	0.00023	3.054239	-0.00027
14	33	-348.6048	0.000095	0.691232	-0.00014
15	35	-348.60575	0.000956	1.267538	-0.00005
16	37	-348.60576	1.355E-6	0.501379	-2E-6

NOTE: GCONV convergence criterion satisfied.

Fit Statistics

-2 Log Likelihood	-697.2
AIC (smaller is better)	-677.2
AICC (smaller is better)	-676.9
BIC (smaller is better)	-653.5

Parameter Estimates

Parameter	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper	Gradient
th1	6.5226	0.1268	78	51.43	<.0001	0.05	6.2701	6.7752	-0.0084
th1add	-0.3745	0.1533	78	-2.44	0.0168	0.05	-0.6796	-0.06937	0.003301
th2	5.8489	0.1263	78	46.31	<.0001	0.05	5.5975	6.1004	0.009137
th2add	-0.3367	0.1527	78	-2.20	0.0304	0.05	-0.6408	-0.03263	-0.00153
th3	76.0886	2.5121	78	30.29	<.0001	0.05	71.0875	81.0898	-0.00364
th3add	0.5558	2.9519	78	0.19	0.8511	0.05	-5.3210	6.4325	0.006411
th4	3.6806	0.4125	78	8.92	<.0001	0.05	2.8594	4.5019	-0.0003
th4add	0.6532	0.5348	78	1.22	0.2256	0.05	-0.4115	1.7180	-0.00006
se2	0.009389	0.000565	78	16.63	<.0001	0.05	0.008265	0.01051	-0.50138
sa2	0.3931	0.06273	78	6.27	<.0001	0.05	0.2682	0.5180	0.001447

```
proc nlmixed data=sitka89;
  title 'OUTPUT # FOUR';
  parms th1=6.6 th1add=-0.4 th2=5.8 th2add=-0.3 th3=76 th4=3.7 se2=.01 sa2=.4;
  th1a=th1+th1add*dumoz+a;
  th2a=th2+th2add*dumoz+a;
  t=(timea/th3)**th4; den=1+t;
  mean=th1a+(th2a-th1a)/den;
  model size~normal(mean,se2);
  random a~normal(0,sa2) subject=tree;
run;
```

OUTPUT # FOUR

The NLMIXED Procedure

Specifications

Data Set	WORK.SITKA89
Dependent Variable	size
Distribution for Dependent Variable	Normal
Random Effects	a
Distribution for Random Effects	Normal
Subject Variable	tree
Optimization Technique	Dual Quasi-Newton
Integration Method	Adaptive Gaussian Quadrature

Dimensions

Observations Used	632
Observations Not Used	0
Total Observations	632
Subjects	79
Max Obs Per Subject	8
Parameters	8

Quadrature Points 1

Parameters

th1	th1add	th2	th2add	th3	th4	se2	sa2	NegLogLike
6.6	-0.4	5.8	-0.3	76	3.7	0.01	0.4	-316.63388

Iteration History

Iter	Calls	NegLogLike	Diff	MaxGrad	Slope
1	7	-317.31375	0.679863	1204.922	-30625.2
2	11	-333.10108	15.78733	2034.599	-11685.2
3	13	-343.52053	10.41945	3184.134	-658.516
4	14	-346.52611	3.005579	1119.373	-8.9261
5	15	-347.11713	0.591028	302.0163	-0.88318
6	17	-347.38318	0.266045	346.3642	-0.59184
7	18	-347.62049	0.237308	595.6719	-0.39761
8	19	-347.69959	0.079099	390.0771	-0.25224
9	20	-347.80138	0.101788	56.54733	-0.21592
10	22	-347.80316	0.001788	6.506249	-0.00356
11	23	-347.80621	0.00305	47.29294	-0.001
12	25	-347.82772	0.021507	152.4459	-0.00803
13	26	-347.85902	0.031296	11.67974	-0.0225
14	28	-347.86121	0.002192	11.32688	-0.00306
15	29	-347.86379	0.002585	8.933038	-0.00099
16	31	-347.86387	0.000073	0.843456	-0.00013
17	33	-347.86387	5.179E-7	0.068732	-1.07E-6

NOTE: GCONV convergence criterion satisfied.

Fit Statistics

-2 Log Likelihood	-695.7
AIC (smaller is better)	-679.7
AICC (smaller is better)	-679.5
BIC (smaller is better)	-660.8

Parameter Estimates

Parameter	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper	Gradient
th1	6.5124	0.1261	78	51.66	<.0001	0.05	6.2614	6.7633	-0.00309
th1add	-0.3594	0.1522	78	-2.36	0.0207	0.05	-0.6623	-0.05649	-0.005
th2	5.8561	0.1261	78	46.45	<.0001	0.05	5.6051	6.1071	0.003259
th2add	-0.3470	0.1524	78	-2.28	0.0255	0.05	-0.6503	-0.04365	0.005346
th3	76.4949	1.3254	78	57.71	<.0001	0.05	73.8563	79.1336	9.902E-6
th4	4.0971	0.2641	78	15.51	<.0001	0.05	3.5713	4.6230	-0.00016
se2	0.009415	0.000566	78	16.63	<.0001	0.05	0.008288	0.01054	0.068732
sa2	0.3931	0.06273	78	6.27	<.0001	0.05	0.2682	0.5179	-0.00031

```
proc nlmixed data=sitka89;
  title 'OUTPUT # Five';
  parms th1=6.6 th12add=-0.4 th2=5.8 th3=76 th4=3.7 se2=.01 sa2=.4;
  th1a=th1+th12add*dumoz+a;
  th2a=th2+th12add*dumoz+a;
  t=(timea/th3)**th4; den=1+t;
  mean=th1a+(th2a-th1a)/den;
  model size~normal(mean,se2);
  random a~normal(0,sa2) subject=tree;
  predict mean out=new;
run;
```

OUTPUT # Five

The NLMIXED Procedure

Specifications

Data Set	WORK.SITKA89
Dependent Variable	size
Distribution for Dependent Variable	Normal
Random Effects	a
Distribution for Random Effects	Normal
Subject Variable	tree
Optimization Technique	Dual Quasi-Newton
Integration Method	Adaptive Gaussian Quadrature

Dimensions

Observations Used	632
Observations Not Used	0
Total Observations	632
Subjects	79
Max Obs Per Subject	8
Parameters	7
Quadrature Points	1

Parameters

th1	th12add	th2	th3	th4	se2	sa2	NegLogLike
6.6	-0.4	5.8	76	3.7	0.01	0.4	-258.29846

Iteration History

Iter	Calls	NegLogLike	Diff	MaxGrad	Slope
1	11	-267.33811	9.039651	968.5474	-563468
2	15	-342.9197	75.58159	2729.23	-20844.7
3	16	-346.63461	3.714915	884.7016	-7.37203
4	17	-346.91999	0.285371	786.6042	-1.13702
5	19	-347.29486	0.374871	19.89597	-0.42069
6	21	-347.46723	0.172375	332.5291	-0.29263
7	22	-347.58013	0.112901	284.4285	-0.29251
8	24	-347.62433	0.044195	77.54163	-0.09538
9	27	-347.6273	0.002969	10.72498	-0.01081
10	29	-347.63381	0.006513	23.89612	-0.00092
11	30	-347.6446	0.010786	4.357682	-0.00773
12	31	-347.65327	0.008668	31.71636	-0.00814
13	33	-347.67209	0.018829	17.66695	-0.01216
14	34	-347.6853	0.013207	20.52766	-0.01529
15	36	-347.68899	0.003687	0.235797	-0.00732
16	38	-347.68899	9.686E-7	0.025742	-1.86E-6

NOTE: GCONV convergence criterion satisfied.

Fit Statistics

-2 Log Likelihood	-695.4
AIC (smaller is better)	-681.4
AICC (smaller is better)	-681.2
BIC (smaller is better)	-664.8

Parameter Estimates

Parameter	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper	Gradient
th1	6.5087	0.1259	78	51.70	<.0001	0.05	6.2581	6.7593	-0.00004
th12add	-0.3541	0.1519	78	-2.33	0.0223	0.05	-0.6565	-0.05173	8.463E-6
th2	5.8610	0.1258	78	46.59	<.0001	0.05	5.6106	6.1114	0.000053
th3	76.4968	1.3252	78	57.72	<.0001	0.05	73.8585	79.1350	-8.25E-6
th4	4.1000	0.2644	78	15.51	<.0001	0.05	3.5737	4.6263	0.000018
se2	0.009421	0.000567	78	16.63	<.0001	0.05	0.008293	0.01055	-0.02574
sa2	0.3931	0.06273	78	6.27	<.0001	0.05	0.2682	0.5179	-0.00027

Plot of residuals versus predicted values by treatment group