Directions: Answer the following six problems in the space given showing all relevant work - please write neatly!

- 1. (2.25 + 2.25 + 2 + 3.5 = 10 points) Replacement times for TV sets are normally distributed with a mean of **8.2** years and a standard deviation of **1.1** years. For this exercise, keep at least 4 decimal places in all your calculations.
 - (a) What is the probability that a randomly selected TV will have a replacement time between 7.0 and 8.5 years?

Answer

(b) If you want to provide a warranty so that only **1%** of the TV sets will be replaced before a warranty expires, what is the time length of the warranty?

Answer _____

(c) What is the probability that a randomly selected TV will have a replacement time more than 9.0 years?

Answer

(d) In a random sample of **four** such TV sets, what is the probability that **exactly one of the four** will have a replacement time **more than 9.0** years (and the other three will have a replacement time of no more than 9.0 years)?

Answer

2. (2.5 + 2 + 2.5 + 3 = 10 Points) In a certain region of the US, the daily rainfall (in inches) is a continuous random variable X with probability density function (pdf)

 $f(x) = (3/4)(2x - x^2)$ for 0 < x < 2 and f(x) = 0 elsewhere.

(a) Find the **cumulative distribution function (cdf)**, showing all work.

(b) Find P(X > 1.5), showing all work.

(c) Find the **expected value of X** showing all work.

Answer _____

Answer

(d) Find the standard deviation (SD) of X showing all work.

3. (2 + 2.5 + 2.5 + 3 = 10 Points) Suppose that the time T (in years) until a PC breaks has the cumulative distribution function (cdf)

 $F(t) = 1 - e^{-2t}$, for t > 0

(a) Find the **probability density function (pdf)** of T. Show all work.

Answer

(b) Find the **expected value of T**. Show all work or give your justification.

Answer

(c) Find the variance of T. Show all work or give your justification.

Answer _____

(d) Given that a PC has not broken in the first year, find the probability that it doesn't break in the following year. Show your work and give your justification.

Answer _____

- 4. (4 + 2 + 4 = 10 Points) Among 16 computer disk drives in stock at a lab, 11 are new whereas 5 have been rebuilt after a previous failure. Two of the disk drives are randomly selected for a networking experiment.
 - (a) Make a table of the probability mass function (pmf) for the number of disk drives selected which are rebuilt.

(b) Find the probability that **at least one** of the selected drives will be rebuilt? Show your calculations.

Answer _____

(c) Find the **mean** and **standard deviation** for the numbers of selected rebuilt drives. Show all calculations.

- 5. (1.5 + 2 + 2 + 2 + 2.5 = 10 points) Suppose that among women under 25 years of age who smoke during their pregnancies, **10%** of them have babies classified as low birth-weight (defined as weighing less than 2.5 kg).
 - (a) Among 20 pregnancies from this population, what is the probability that there will be **no (zero)** low birth-weight babies? Show all calculations or give table page number.

Answer _____

(b) Among 20 pregnancies from this population, what is the probability that there will be at least 2 low birth-weight babies? Show all calculations or give table page number.

Answer

(c) Among 20 pregnancies from this population, use the *Poisson approximation* to approximate the probability that there will be **at least 2** low birth-weight babies? Show all calculations.

Answer

(d) In repeatedly sampling from this population of women, suppose that in the first twenty trials no low birth-weight babies were observed. What is the probability that the **first** low birth-weight baby occurs on the 25th trial?

Answer

(e) Among **500** pregnancies from this population, use the *Normal approximation with the continuity correction* to approximate the probability that **at least 40** babies are classified as low birth-weight.

Answer

6. (10 Points) To test the flammability of a certain type of synthetic fiber, **17** batches of the fiber were subjected to a flame. The following are the ignition times recorded to the nearest tenth of a second. Test whether the median ignition time is **greater than 2.6**. Use the *sign test* at the **0.10** level of significance.

3.2	2.1	3.5	3.7	3.0	2.3	3.3	3.9	3.6	1.8	2.7	3.4	2.6	3.4	2.8	1.0	2.4
Null I	Null Hypothesis (H ₀):															
Alter	native	Hypoth	nesis (H	[_A):												
Alpha =																
Test s	statistic	2														
P-Val	lue =															
Decision is to (circle correct choice):							Reject H	H ₀	Retai	n H ₀	Ne	ither o	f these	(state	why)	
State	State your decision in plain English in context of the problem															