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RISK AND INSURANCE

by

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The Education and Examination Committee provides study notes to persons preparing for the examinations of the Society of Actuaries. They are intended to acquaint candidates with some of the theoretical and practical considerations involved in the various subjects. While varying opinions are presented where appropriate, limits on the length of the material and other considerations sometimes prevent the inclusion of all possible opinions. These study notes do not, however, represent any official opinion, interpretations or endorsement of the Society of Actuaries or its Education and Examination Committee. The Society is grateful to the authors for their contributions in preparing the study notes.

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RISK AND INSURANCE

I. INTRODUCTION

People seek security. A sense of security may be the next basic goal after food, clothing, and shelter. An individual with economic security is fairly certain that he can satisfy his needs (food, shelter, medical care, and so on) in the present and in the future. Economic risk (which we will refer to simply as risk) is the possibility of losing economic security. Most economic risk derives from variation from the expected outcome.

One measure of risk, used in this study note, is the standard deviation of the possible outcomes. As an example, consider the cost of a car accident for two different cars, a Porsche and a Toyota. In the event of an accident the expected value of repairs for both cars is 2500. However, the standard deviation for the Porsche is 1000 and the standard deviation for the Toyota is 400. If the cost of repairs is normally distributed, then the probability that the repairs will cost more than 3000 is 31% for the Porsche but only 11% for the Toyota.

Modern society provides many examples of risk. A homeowner faces a large potential for variation associated with the possibility of economic loss caused by a house fire. A driver faces a potential economic loss if his car is damaged. A larger possible economic risk exists with respect to potential damages a driver might have to pay if he injures a third party in a car accident for which he is responsible.

Historically, economic risk was managed through informal agreements within a defined community. If someone's barn burned down and a herd of milking cows was destroyed, the community would pitch in to rebuild the barn and to provide the farmer with enough cows to replenish the milking stock. This cooperative (*pooling*) concept became formalized in the insurance industry. Under a formal insurance arrangement, each insurance policy purchaser (*policyholder*) still implicitly pools his risk with all other policyholders. However, it is no longer necessary for any individual policyholder to know or have any direct connection with any other policyholder.

II. HOW INSURANCE WORKS

Insurance is an agreement where, for a stipulated payment called the *premium*, one party (the insurer) agrees to pay to the other (the policyholder or his designated beneficiary) a defined amount (the *claim payment* or *benefit*) upon the occurrence of a specific loss. This defined claim payment amount can be a fixed amount or can reimburse all or a part of the loss that occurred. The insurer considers the losses expected for the insurance pool and the potential for variation in order to charge premiums that, in total, will be sufficient to cover all of the projected claim payments for the insurance pool. The premium charged to each of the pool participants is that participant's share of the total premium for the pool. Each premium may be adjusted to reflect any special characteristics of the particular policy. As will be seen in the next section, the larger the policy pool, the more predictable its results.

Normally, only a small percentage of policyholders suffer losses. Their losses are paid out of the premiums collected from the pool of policyholders. Thus, the entire pool compensates the unfortunate few. Each policyholder exchanges an unknown loss for the payment of a known premium.

Under the formal arrangement, the party agreeing to make the claim payments is the insurance company or the *insurer*. The pool participant is the policyholder. The payments that the policyholder makes to the insurer are premiums. The insurance contract is the policy. The risk of any unanticipated losses is transferred from the policyholder to the insurer who has the right to specify the rules and conditions for participating in the insurance pool.

The insurer may restrict the particular kinds of losses covered. For example, a *peril* is a potential cause of a loss. Perils may include fires, hurricanes, theft, and heart attack. The insurance policy may define specific perils that are covered, or it may cover all perils with certain named exclusions (for example, loss as a result of war or loss of life due to suicide).

Hazards are conditions that increase the probability or expected magnitude of a loss. Examples include smoking when considering potential healthcare losses, poor wiring in a house when considering losses due to fires, or a California residence when considering earthquake damage.

In summary, an insurance contract covers a policyholder for economic loss caused by a peril named in the policy. The policyholder pays a known premium to have the insurer guarantee payment for the unknown loss. In this manner, the policyholder transfers the economic risk to the insurance company. Risk, as discussed in Section I, is the variation in potential economic outcomes. It is measured by the variation between possible outcomes and the expected outcome: the greater the standard deviation, the greater the risk.

III. A MATHEMATICAL EXPLANATION

Losses depend on two random variables. The first is the number of losses that will occur in a specified period. For example, a healthy policyholder with hospital insurance will have no losses in most years, but in some years he could have one or more accidents or illnesses requiring hospitalization. This random variable for the number of losses is commonly referred to as the *frequency of loss* and its probability distribution is called the *frequency distribution*. The second random variable is the amount of the loss, given that a loss has occurred. For example, the hospital charges for an overnight hospital stay would be much lower than the charges for an extended hospitalization. The amount of loss is often referred to as the *severity* and the probability distribution for the amount of loss is called the *severity distribution*. By combining the frequency distribution with the severity distribution we can determine the overall loss distribution.

Example: *Consider a car owner who has an* 80% *chance of no accidents in a year, a* 20% *chance of being in a single accident in a year, and no chance of being in more than one accident* *in a year. For simplicity, assume that there is a* 50% *probability that after the accident the car will need repairs costing* 500*, a* 40% *probability that the repairs will cost* 5000*, and a* 10% *probability that the car will need to be replaced, which will cost* 15,000*. Combining the frequency and severity distributions forms the following distribution of the random variable X, loss due to accident:*

$$f(x) = \begin{cases} 0.80 & x = 0 \\ 0.10 & x = 500 \\ 0.08 & x = 5000 \\ 0.02 & x = 15{,}000 \end{cases}$$

The car owner's expected loss is the mean of this distribution, $E[X]$*:*

$$E[X] = \sum x \cdot f(x) = 0.80 \cdot 0 + 0.10 \cdot 500 + 0.08 \cdot 5000 + 0.02 \cdot 15{,}000 = 750$$

On average, the car owner spends 750 *on repairs due to car accidents. A* 750 *loss may not seem like much to the car owner, but the possibility of a* 5000 *or* 15,000 *loss could create real concern.*

To measure the potential variability of the car owner's loss, consider the standard deviation of the loss distribution:

$$\begin{aligned} \sigma_X^2 &= \sum \left(x - E[X]\right)^2 f(x) \\ &= 0.80 \cdot (-750)^2 + 0.10 \cdot (-250)^2 + 0.08 \cdot (4250)^2 + 0.02 \cdot (14{,}250)^2 = 5{,}962{,}500 \\ \sigma_X &= \sqrt{5{,}962{,}500} = 2442 \end{aligned}$$

If we look at a particular individual, we see that there can be an extremely large variation in possible outcomes, each with a specific economic consequence. By purchasing an insurance policy, the individual transfers this risk to an insurance company in exchange for a fixed premium. We might conclude, therefore, that if an insurer sells *n* policies to *n* individuals, it assumes the total risk of the *n* individuals. In reality, the risk assumed by the insurer is smaller in total than the sum of the risks associated with each individual policyholder. These results are shown in the following theorem.

Theorem: Let $X_1, X_2, \ldots, X_n$ be independent random variables such that each X_i has an expected value of μ and variance of σ^2. Let $S_n = X_1 + X_2 + \ldots + X_n$. Then:

$$E[S_n] = n \cdot E[X_i] = n\mu \text{, and}$$

$$Var[S_n] = n \cdot Var[X_i] = n \cdot \sigma^2 .$$

The standard deviation of S_n is $\sqrt{n} \cdot \sigma$, which is less than $n\sigma$, the sum of the standard deviations for each policy.

Furthermore, the *coefficient of variation*, which is the ratio of the standard deviation to the mean, is $\dfrac{\sqrt{n} \cdot \sigma}{n \cdot \mu} = \dfrac{\sigma}{\sqrt{n} \cdot \mu}$. This is smaller than $\dfrac{\sigma}{\mu}$, the coefficient of variation for each individual X_i.

The coefficient of variation is useful for comparing variability between positive distributions with different expected values. So, given *n* independent policyholders, as *n* becomes very large, the insurer's risk, as measured by the coefficient of variation, tends to zero.

Example: *Going back to our example of the car owner, consider an insurance company that will reimburse repair costs resulting from accidents for* 100 *car owners, each with the same risks as in our earlier example. Each car owner has an expected loss of* 750 *and a standard deviation of* 2442. *As a group the expected loss is* 75,000 *and the variance is* 596,250,000. *The standard deviation is* $\sqrt{596{,}250{,}000} = 24{,}418$ *which is significantly less than the sum of the standard deviations,* 244,182. *The ratio of the standard deviation to the expected loss is* $24{,}418/75{,}000 = 0.326$, *which is significantly less than the ratio of* $2442/750 = 3.26$ *for one car owner.*

It should be clear that the existence of a private insurance industry in and of itself does not decrease the frequency or severity of loss. Viewed another way, merely entering into an insurance contract does not change the policyholder's expectation of loss. Thus, given perfect information, the amount that any policyholder should have to pay an insurer equals the expected claim payments plus an amount to cover the insurer's expenses for selling and servicing the policy, including some profit. The expected amount of claim payments is called the *net premium* or *benefit premium*. The term *gross premium* refers to the total of the net premium and the amount to cover the insurer's expenses and a margin for unanticipated claim payments.

Example: *Again considering the* 100 *car owners, if the insurer will pay for all of the accident-related car repair losses, the insurer should collect a premium of at least* 75,000 *because that is the expected amount of claim payments to policyholders. The net premium or benefit premium would amount to* 750 *per policy. The insurer might charge the policyholders an additional* 30% *so that there would be* 22,500 *to help the insurer pay expenses related to the insurance policies and cover any unanticipated claim payments. In this case* $750\theta 130\%=975$ *would be the gross premium for a policy.*

Policyholders are willing to pay a gross premium for an insurance contract, which exceeds the expected value of their losses, in order to substitute the fixed, zero-variance premium payment for an unmanageable amount of risk inherent in not insuring.

IV. CHARACTERISTICS OF AN INSURABLE RISK

We have stated previously that individuals see the purchase of insurance as economically advantageous. The insurer will agree to the arrangement if the risks can be pooled, but will need some safeguards. With these principles in mind, what makes a risk insurable? What kinds of risk would an insurer be willing to insure?

The potential loss must be significant and important enough that substituting a known insurance premium for an unknown economic outcome (given no insurance) is desirable.

The loss and its economic value must be well-defined and out of the policyholder's control. The policyholder should not be allowed to cause or encourage a loss that will lead to a benefit or claim payment. After the loss occurs, the policyholder should not be able to unfairly adjust the value of the loss (for example, by lying) in order to increase the amount of the benefit or claim payment.

Covered losses should be reasonably independent. The fact that one policyholder experiences a loss should not have a major effect on whether other policyholders do. For example, an insurer would not insure all the stores in one area against fire, because a fire in one store could spread to the others, resulting in many large claim payments to be made by the insurer.

These criteria, if fully satisfied, mean that the risk is insurable. The fact that a potential loss does not fully satisfy the criteria does not necessarily mean that insurance will not be issued, but some special care or additional risk sharing with other insurers may be necessary.

V. EXAMPLES OF INSURANCE

Some readers of this note may already have used insurance to reduce economic risk. In many places, to drive a car legally, you must have liability insurance, which will pay benefits to a person that you might injure or for property damage from a car accident. You may purchase collision insurance for your car, which will pay toward having your car repaired or replaced in case of an accident. You can also buy coverage that will pay for damage to your car from causes other than collision, for example, damage from hailstones or vandalism.

Insurance on your residence will pay toward repairing or replacing your home in case of damage from a covered peril. The contents of your house will also be covered in case of damage or theft. However, some perils may not be covered. For example, flood damage may not be covered if your house is in a floodplain.

At some point, you will probably consider the purchase of life insurance to provide your family with additional economic security should you die unexpectedly. Generally, life insurance provides for a fixed benefit at death. However, the benefit may vary over time. In addition, the length of the premium payment period and the period during which a death is eligible for a benefit may each vary. Many combinations and variations exist.

When it is time to retire, you may wish to purchase an *annuity* that will provide regular income to meet your expenses. A basic form of an annuity is called a life annuity, which pays a regular amount for as long as you live. Annuities are the complement of life insurance. Since payments are made until death, the peril is survival and the risk you have shifted to the insurer is the risk of living longer than your savings would last. There are also annuities that combine the basic life annuity with a benefit payable upon death. There are many different forms of death benefits that can be combined with annuities.

Disability income insurance replaces all or a portion of your income should you become disabled. Health insurance pays benefits to help offset the costs of medical care, hospitalization, dental care, and so on.

Employers may provide many of the insurance coverages listed above to their employees.

VI. LIMITS ON POLICY BENEFITS

In all types of insurance there may be limits on benefits or claim payments. More specifically, there may be a maximum limit on the total reimbursed; there may be a minimum limit on losses that will be reimbursed; only a certain percentage of each loss may be reimbursed; or there may be different limits applied to particular types of losses.

In each of these situations, the insurer does not reimburse the entire loss. Rather, the policyholder must cover part of the loss himself. This is often referred to as *coinsurance*.

The next two sections discuss specific types of limits on policy benefits.

DEDUCTIBLES

A policy may stipulate that losses are to be reimbursed only in excess of a stated threshold amount, called a *deductible*. For example, consider insurance that covers a loss resulting from an accident but includes a 500 deductible. If the loss is less than 500 the insurer will not pay anything to the policyholder. On the other hand, if the loss is more than 500, the insurer will pay for the loss in excess of the deductible. In other words, if the loss is 2000, the insurer will pay 1500. Reasons for deductibles include the following:

(1) Small losses do not create a claim payment, thus saving the expenses of processing the claim.
(2) Claim payments are reduced by the amount of the deductible, which is translated into premium savings.
(3) The deductible puts the policyholder at risk and, therefore, provides an economic incentive for the policyholder to prevent losses that would lead to claim payments.

Problems associated with deductibles include the following:

(1) The policyholder may be disappointed that losses are not paid in full. Certainly, deductibles increase the risk for which the policyholder remains responsible.
(2) Deductibles can lead to misunderstandings and bad public relations for the insurance company.
(3) Deductibles may make the marketing of the coverage more difficult for the insurance company.
(4) The policyholder may overstate the loss to recover the deductible.

Note that if there is a deductible, there is a difference between the value of a loss and the associated claim payment. In fact, for a very small loss there will be no claim payment. Thus, it is essential to differentiate between losses and claim payments as to both frequency and severity.

Example: *Consider the group of* 100 *car owners that was discussed earlier. If the policy provides for a* 500 *deductible, what would the expected claim payments and the insurer's risk be?*

The claim payment distribution for each policy would now be:

$$f(y) = \begin{cases} 0.90 & loss = 0 \text{ or } 500 \quad y = 0 \\ 0.08 & loss = 5000 \quad y = 4500 \\ 0.02 & loss = 15{,}000 \quad y = 14{,}500 \end{cases}$$

The expected claim payments and standard deviation for one policy would be:

$$E[Y] = 0.90 \cdot 0 + 0.08 \cdot 4500 + 0.02 \cdot 14{,}500 = 650$$
$$\sigma_Y^2 = 0.90 \cdot (-650)^2 + 0.08 \cdot (3850)^2 + 0.02 \cdot (13{,}850)^2 = 5{,}402{,}500$$
$$\sigma_Y = \sqrt{5{,}402{,}500} = 2324$$

The expected claim payments for the hundred policies would be 65,000*, the variance would be* 540,250,000 *and the standard deviation would be* 23,243.

As shown in this example, the presence of the deductible will save the insurer from having to process the relatively small claim payments of 500*. The probability of a claim occurring drops from 20% to 10% per policy. The deductible lowers the expected claim payments for the hundred policies from* 75,000 *to* 65,000 *and the standard deviation will fall from* 24,418 *to* 23,243.

BENEFIT LIMITS

A benefit limit sets an upper bound on how much the insurer will pay for any loss. Reasons for placing a limit on the benefits include the following:

(1) The limit prevents total claim payments from exceeding the insurer's financial capacity.
(2) In the context of risk, an upper bound to the benefit lessens the risk assumed by the insurer.
(3) Having different benefit limits allows the policyholder to choose appropriate coverage at an appropriate price, since the premium will be lower for lower benefit limits.

In general, the lower the benefit limit, the lower the premium. However, in some instances the premium differences are relatively small. For example, an increase from 1 million to 2 million liability coverage in an auto policy would result in a very small increase in premium. This is because losses in excess of 1 million are rare events, and the premium determined by the insurer is based primarily on the expected value of the claim payments.

As has been implied previously, a policy may have more than one limit, and, overall, there is more than one way to provide limits on benefits. Different limits may be set for different perils. Limits might also be set as a percentage of total loss. For example, a health insurance policy may pay healthcare costs up to 5000, and it may only reimburse for 80% of these costs. In this case, if costs were 6000, the insurance would reimburse 4000, which is 80% of the lesser of 5000 and the actual cost.

Example: *Looking again at the* 100 *insured car owners, assume that the insurer has not only included a* 500 *deductible but has also placed a maximum on a claim payment of* 12,500. *What would the expected claim payments and the insurer's risk be?*

The claim payment distribution for each policy would now be:

$$f(y) = \begin{cases} 0.90 & loss = 0 \ or \ 500 & y = 0 \\ 0.08 & loss = 5000 & y = 4500 \\ 0.02 & loss = 15{,}000 & y = 12{,}500 \end{cases}$$

The expected claim payments and standard deviation for one policy would be:

$$E[Y] = 0.90 \cdot 0 + 0.08 \cdot 4500 + 0.02 \cdot 12{,}500 = 610$$
$$\sigma_Y^2 = 0.90 \cdot (-610)^2 + 0.08 \cdot (3890)^2 + 0.02 \cdot (11{,}890)^2 = 4{,}372{,}900$$
$$\sigma_Y = \sqrt{4{,}372{,}900} = 2091$$

The expected claim payments for the hundred policies would be 61,000*, the variance would be* 437,290,000, *and the standard deviation would be* 20,911.

In this case, the presence of the deductible and the benefit limit lowers the insurer's expected claim payments for the hundred policies from 75,000 *to* 61,000 *and the standard deviation will fall from* 24,418 *to* 20,911.

VII. INFLATION

Many insurance policies pay benefits based on the amount of loss at existing price levels. When there is price inflation, the claim payments increase accordingly. However, many deductibles and benefit limits are expressed in fixed amounts that do not increase automatically as inflation increases claim payments. Thus, the impact of inflation is altered when deductibles and other limits are not adjusted.

Example: *Looking again at the* 100 *insured car owners with a* 500 *deductible and no benefit limit, assume that there is* 10% *annual inflation. Over the next 5 years, what would the expected claim payments and the insurer's risk be?*

Because of the 10% *annual inflation in new car and repair costs, a* 5000 *loss in year 1 will be equivalent to a loss of* 5000θ1.10=5500 *in year 2; a loss of* $5000\theta(1.10)^2$=6050 *in year 3; and a loss of* $5000\theta(1.10)^3$=6655 *in year 4.*

The claim payment distributions, expected losses, expected claim payments, and standard deviations for each policy are:

Policy with a 500 Deductible

	f(y,t)	0.80	0.10	0.08	0.02	Expected Amount	Standard Deviation
Year 1							
	Loss	*0*	*500*	*5000*	*15,000*	*750*	
	Claim	*0*	*0*	*4500*	*14,500*	*650*	*2324*
Year 2							
	Loss	*0*	*550*	*5500*	*16,500*	*825*	
	Claim	*0*	*50*	*5000*	*16,000*	*725*	*2568*
Year 3							
	Loss	*0*	*605*	*6050*	*18,150*	*908*	
	Claim	*0*	*105*	*5550*	*17,650*	*808*	*2836*
Year 4							
	Loss	*0*	*666*	*6655*	*19,965*	*998*	
	Claim	*0*	*166*	*6155*	*19,465*	*898*	*3131*
Year 5							
	Loss	*0*	*732*	*7321*	*21,962*	*1098*	
	Claim	*0*	*232*	*6821*	*21,462*	*998*	*3456*

Looking at the increases from one year to the next, the expected losses increase by 10% *each year but the expected claim payments increase by more than* 10% *annually. For example, expected losses grow from* 750 *in year 1 to* 1098 *in year 5, an increase of* 46%. *However, expected claim payments grow from* 650 *in year 1 to* 998 *in year 5, an increase of* 54%. *Similarly, the standard deviation of claim payments also increases by more than* 10% *annually. Both phenomena are caused by a deductible that does not increase with inflation.*

Next, consider the effect of inflation if the policy also has a limit setting the maximum claim payment at 12,500.

Policy with a Deductible of 500 and Maximum Claim Payment of 12,500

						Expected Amount	Standard Deviation
	f(y,t)	*0.80*	*0.10*	*0.08*	*0.02*		
Year 1							
	Loss	*0*	*500*	*5000*	*15,000*	*750*	
	Claim	*0*	*0*	*4500*	*12,500*	*610*	*2091*
Year 2							
	Loss	*0*	*550*	*5500*	*16,500*	*825*	
	Claim	*0*	*50*	*5000*	*12,500*	*655*	*2167*
Year 3							
	Loss	*0*	*605*	*6050*	*18,150*	*908*	
	Claim	*0*	*105*	*5550*	*12,500*	*705*	*2257*
Year 4							
	Loss	*0*	*666*	*6655*	*19,965*	*998*	
	Claim	*0*	*166*	*6155*	*12,500*	*759*	*2363*
Year 5							
	Loss	*0*	*732*	*7321*	*21,962*	*1098*	
	Claim	*0*	*232*	*6821*	*12,500*	*819*	*2486*

A fixed deductible with no maximum limit exaggerates the effect of inflation. Adding a fixed maximum on claim payments limits the effect of inflation. Expected claim payments grow from 610 *in year 1 to* 819 *in year 5, an increase of* 34%, *which is less than the* 46% *increase in expected losses. Similarly, the standard deviation of claim payments increases by less than the 10% annual increase in the standard deviation of losses. Both phenomena occur because the benefit limit does not increase with inflation.*

VIII. A CONTINUOUS SEVERITY EXAMPLE

In the car insurance example, we assumed that repair or replacement costs could take only a fixed number of values. In this section we repeat some of the concepts and calculations introduced in prior sections but in the context of a continuous severity distribution.

Consider an insurance policy that reimburses annual hospital charges for an insured individual. The probability of any individual being hospitalized in a year is 15%. *That is,* $P(H=1) = 0.15$. *Once an individual is hospitalized, the charges X have a probability density function (p.d.f.)* $f_X\left(x \middle| H=1\right) = 0.1e^{-0.1x}$ for $x > 0$.

Determine the expected value, the standard deviation, and the ratio of the standard deviation to the mean (coefficient of variation) of hospital charges for an insured individual.

The expected value of hospital charges is:

$$E[X] = P(H \neq 1)E[X|H \neq 1] + P(H = 1)E[X|H = 1]$$

$$= 0.85 \cdot 0 + 0.15 \int_0^\infty 0.1\, x \cdot e^{-0.1x} dx \;=\; -0.15\, x \cdot e^{-0.1x} \Big|_0^\infty + 0.15 \int_0^\infty e^{-0.1x} dx$$

$$= -0.15 \cdot 10 \cdot e^{-0.1x} \Big|_0^\infty = 1.5$$

$$E[X^2] = P(H \neq 1)E[X^2|H \neq 1] + P(H = 1)E[X^2|H = 1]$$

$$= 0.85 \cdot 0^2 + 0.15 \int_0^\infty 0.1\, x^2 \cdot e^{-0.1x} dx$$

$$= -0.15\, x^2 \cdot e^{-0.1x} \Big|_0^\infty + 0.15 \cdot 10 \int_0^\infty 0.1 \cdot 2x \cdot e^{-0.1x} dx \;=\; 30$$

The variance is: $\sigma_X^2 = E[X^2] - (E[X])^2 = 30 - (1.5)^2 = 27.75$

The standard deviation is: $\sigma_X = \sqrt{27.75} = 5.27$

The coefficient of variation is: $\sigma_X / E[X] = 5.27 / 1.5 = 3.51$

An alternative solution would recognize and use the fact that $f_X(X|H = 1)$ is an exponential distribution to simplify the calculations.

Determine the expected claim payments, standard deviation and coefficient of variation for an insurance pool that reimburses hospital charges for 200 *individuals. Assume that claims for each individual are independent of the other individuals.*

Let $S = \sum_{i=1}^{200} X_i$

$$E[S] = 200\, E[X] \;=\; 300$$

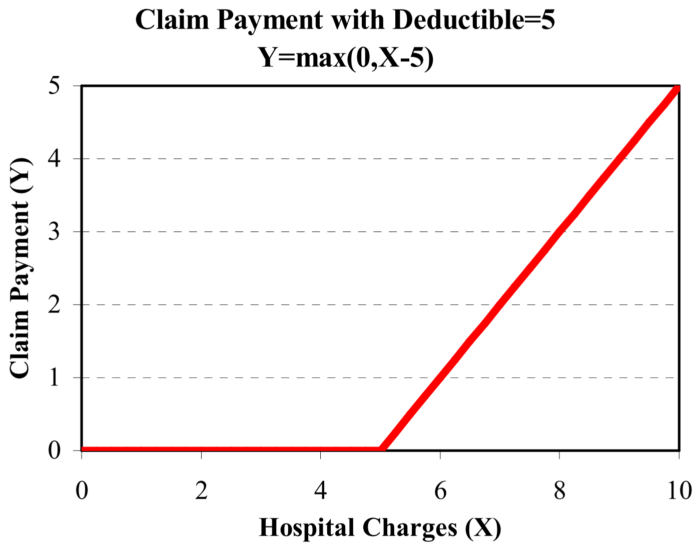
$$\sigma_S^2 = 200\, \sigma_X^2 = 5550$$

$$\sigma_S = \sqrt{200}\, \sigma_X = 74.50$$

Coefficient of variation $= \sigma_S / E[S] = 0.25$

If the insurer includes a deductible of 5 on annual claim payments for each individual, what would the expected claim payments and the standard deviation be for the pool?

The relationship of claim payments to hospital charges is shown in the graph below:



There are three different cases to consider for an individual:

(1) There is no hospitalization and thus no claim payments.

(2) There is hospitalization, but the charges are less than the deductible.

(3) There is hospitalization and the charges are greater than the deductible.

In the third case, the p.d.f. of claim payments is:

$$f_Y(y|X>5, H=1) = \frac{f_X(y+5|H=1)}{P(X>5|H=1)} = \frac{0.1\, e^{-0.1(y+5)}}{P(X>5|H=1)}$$

Summing the three cases:

$$E[Y] = P(H \neq 1)E[Y|H \neq 1] + P(X \leq 5, H=1)E[Y|X \leq 5, H=1] + P(X>5, H=1)E[Y|X>5, H=1]$$

$$= P(H \neq 1)\cdot 0 + P(H=1)\cdot P(X \leq 5|H=1)\cdot 0 + P(H=1)\cdot P(X>5|H=1)\cdot E[Y|X>5, H=1]$$

$$= 0.15\int_0^\infty 0.1\, y\cdot e^{-0.1(y+5)}dy \;=\; 0.15\cdot e^{-0.5}\int_0^\infty 0.1\, y\cdot e^{-0.1y}dy$$

$$= 0.15\cdot e^{-0.5}\cdot 10 = 0.91$$

$$E\left[Y^2\right] = P\left(H \neq 1\right)\cdot 0^2 + P\left(H = 1\right)\cdot P\left(X \leq 5 \middle| H = 1\right)\cdot 0^2 + 0.15\int_0^\infty 0.1y^2 \cdot e^{-0.1(y+5)}dy$$

$$= 0.15\cdot e^{-0.5}\int_0^\infty 0.1y^2 \cdot e^{-0.1y}dy$$

$$= 30e^{-0.5} = 18.20$$

$$\sigma_Y^2 = 18.20 - \left(0.91\right)^2 = 17.37$$

$$\sigma_Y = \sqrt{17.37} = 4.17$$

For the pool of 200 individuals, let $S_Y = \sum_{i=1}^{200} Y_i$

$$E\left[S_Y\right] = 200\ E\left[Y\right] = 182$$

$$\sigma_{S_Y}^2 = 200\ \sigma_Y^2 = 3474$$

$$\sigma_{S_Y} = \sqrt{200}\ \sigma_Y = 58.94$$

Assume further that the insurer only reimburses 80% *of the charges in excess of the* 5 *deductible. What would the expected claim payments and the standard deviation be for the pool?*

$$E\left[80\%\cdot S_Y\right] = 0.8\cdot E\left[S_Y\right] = 146$$

$$\sigma_{80\% S_Y}^2 = \left(0.8\right)^2\cdot\sigma_{S_Y}^2 = 2223$$

$$\sigma_{80\% S_Y} = 0.8\cdot\sigma_{S_Y} = 47.15$$

IX. THE ROLE OF THE ACTUARY

This study note has outlined some of the fundamentals of insurance. Now the question is: what is the role of the actuary?

At the most basic level, actuaries have the mathematical, statistical and business skills needed to determine the expected costs and risks in any situation where there is financial uncertainty and data for creating a model of those risks. For insurance, this includes developing net premiums (benefit premiums), gross premiums, and the amount of assets the insurer should have on hand to assure that benefits and expenses can be paid as they arise.

The actuary would begin by trying to estimate the frequency and severity distribution for a particular insurance pool. This process usually begins with an analysis of past experience. The actuary will try to use data gathered from the insurance pool or from a group as similar to the insurance pool as possible. For instance, if a group of active workers were being insured for healthcare expenditures, the actuary would not want to use data that included disabled or retired individuals.

In analyzing past experience, the actuary must also consider how reliable the past experience is as a predictor of the future. Assuming that the experience collected is representative of the insurance pool, the more data, the more assurance that it will be a good predictor of the true underlying probability distributions. This is illustrated in the following example:

An actuary is trying to determine the underlying probability that a 70-year-old woman will die within one year. The actuary gathers data using a large random sample of 70-year-old women from previous years and identifies how many of them died within one year. The probability is estimated by the ratio of the number of deaths in the sample to the total number of 70-year-old women in the sample. The Central Limit Theorem tells us that if the underlying distribution has a mean of p *and standard deviation of* σ *then the mean of a large random sample of size n is approximately normally distributed with mean* p *and standard deviation* $\frac{\sigma}{\sqrt{n}}$. *The larger the size of the sample, the smaller the variation between the sample mean and the underlying value of* p.

When evaluating past experience the actuary must also watch for fundamental changes that will alter the underlying probability distributions. For example, when estimating healthcare costs, if new but expensive techniques for treatment are discovered and implemented then the distribution of healthcare costs will shift up to reflect the use of the new techniques.

The frequency and severity distributions are developed from the analysis of the past experience and combined to develop the loss distribution. The claim payment distribution can then be derived by adjusting the loss distribution to reflect the provisions in the policies, such as deductibles and benefit limits.

If the claim payments could be affected by inflation, the actuary will need to estimate future inflation based on past experience and information about the current state of the economy. In the case of insurance coverages where today's premiums are invested to cover claim payments in the years to come, the actuary will also need to estimate expected investment returns.

At this point the actuary has the tools to determine the net premium.

The actuary can use similar techniques to estimate a sufficient margin to build into the gross premium in order to cover both the insurer's expenses and a reasonable level of unanticipated claim payments.

Aside from establishing sufficient premium levels for future risks, actuaries also use their skills to determine whether the insurer's assets on hand are sufficient for the risks that the insurer has already committed to cover. Typically this involves at least two steps. The first is to estimate the current amount of assets necessary for the particular insurance pool. The second is to estimate the flow of claim payments, premiums collected, expenses and other income to assure that at each point in time the insurer has enough cash (as opposed to long-term investments) to make the payments.

Actuaries will also do a variety of other projections of the insurer's future financial situation under given circumstances. For instance, if an insurer is considering offering a new kind of policy, the actuary will project potential profit or loss. The actuary will also use projections to assess potential difficulties before they become significant.

These are some of the common actuarial projects done for businesses facing risk. In addition, actuaries are involved in the design of new financial products, company management and strategic planning.

X. CONCLUSION

This study note is an introduction to the ideas and concepts behind actuarial work. The examples have been restricted to insurance, though many of the concepts can be applied to any situation where uncertain events create financial risks.

Later Casualty Actuarial Society and Society of Actuaries exams cover topics including: adjustment for investment earnings; frequency models; severity models; aggregate loss models; survival models; fitting models to actual data; and the credibility that can be attributed to past data. In addition, both societies offer courses on the nature of particular perils and related business issues that need to be considered.